Приклади знаходження похідних

1)

$$y = x^3 - 3x^2 - \frac{1}{6}x^{-6} + 5;$$

$$y' = \left(x^3 - 3x^2 - \frac{1}{6}x^{-6} + 5\right)' = \left(x^3\right)' - 3\left(x^2\right)' - \frac{1}{6}\left(x^{-6}\right)' + 5' =$$

$$= 3x^{3-1} - 3\cdot 2x^{2-1} - \frac{1}{6}\cdot\left(-6\right)x^{-6-1} + 0 = 3x^2 - 6x + x^{-7}.$$

2)

$$y = x^8 - \sqrt{x} + \frac{3}{\sqrt[3]{x^5}} - \frac{1}{x^3};$$

$$y = x^8 - \sqrt{x} + \frac{3}{\sqrt[3]{x^5}} - \frac{1}{x^3} = x^8 - x^{\frac{1}{2}} + 3x^{\frac{5}{3}} - x^{-3}.$$

$$y' = 8x^{8-1} - \frac{1}{2}x^{\frac{1}{2}-1} + 3\cdot\frac{5}{3}x^{\frac{5}{3}-1} - \left(-3\right)x^{-3-1} =$$

$$= 8x^7 - \frac{1}{2\sqrt{x}} + 5\sqrt[3]{x^2} + \frac{3}{x^4}.$$

3)

$$y = \frac{2x^2}{x^2 - 5};,$$

$$y' = \left(\frac{2x^2}{x^2 - 5}\right)' = \frac{\left(2x^2\right)'\left(x^2 - 5\right) - 2x^2\left(x^2 - 5\right)'}{\left(x^2 - 5\right)^2} =$$

$$= \frac{4x\left(x^2 - 5\right) - 2x^2\cdot 2x}{\left(x^2 - 5\right)^2} = \frac{4x}{\left(x^2 - 5\right)} - \frac{4x^3}{\left(x^2 - 5\right)^2}.$$

4)

$$y = \sqrt[3]{3+x^5};$$

$$y' = \left(\sqrt[3]{3+x^5}\right)' = \frac{1}{3}\left(3+x^5\right)^{\frac{1}{3}-1}\cdot\left(3+x^5\right)' =$$

$$= \frac{1}{3}\left(3+x^5\right)^{-\frac{2}{3}}\cdot 5x^{5-1} = \frac{5x^4}{\sqrt[3]{\left(3+x^5\right)^2}}.$$

5)

$$y = \left(x^3+7x^2-3\right)\sqrt{2+x^2};$$

$$y' = \left(x^3+7x^2-3\right)'\sqrt{2+x^2} + \left(x^3+7x^2-3\right)\left(\sqrt{2+x^2}\right)' =$$

$$= \left(3x^{3-1}+7\cdot 2x^{2-1}\right)\sqrt{2+x^2} + \left(x^3+7x^2-3\right)\frac{\left(2+x^2\right)'}{2\sqrt{2+x^2}} =$$

$$= \left(3x^2+14x\right)\sqrt{2+x^2} + \left(x^3+7x^2-3\right)\frac{x}{\sqrt{2+x^2}}.$$

6)

$$y = 2\sin\left(2x^3\right);$$

$$y' = 2\cos\left(2x^3\right)\cdot\left(2x^3\right)' = 2\cos\left(2x^3\right)\cdot 2\cdot 3x^{3-1} =$$

$$= 12x^2\cos\left(2x^3\right)\cdot 2\cdot 3x^{3-1}.$$

7)

$$y = \left(3x^3-5x^2+6\right)^2.$$

$$y' = 2\left(3x^3-5x^2+6\right)^{2-1}\cdot\left(3x^3-5x^2+6\right)' =$$

$$= 2\left(3x^3-5x^2+6\right)\cdot\left(3\cdot 3x^{3-1}-5\cdot 2x^{2-1}\right) =$$

$$= 2x\left(3x^3-5x^2+6\right)\cdot\left(9x-10\right).$$