Lecture 2

THE RELATIO AMONG INTERNAL FORCES AND TENSIONS IN CASE OF TENSION OR COMPRESSION OF BAR

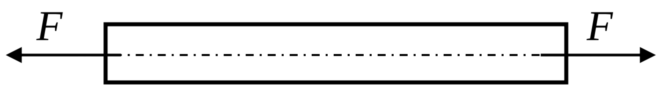
Plan

1. Internal effects of force.
2. Mechanical properties of materials.
3. General form of Hooke's law.

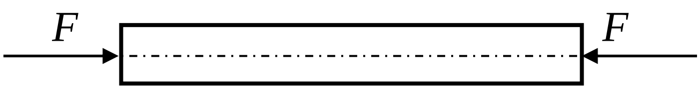
2.1. Internal effects of force

We shall be concerned with what might be called the internal effects of forces acting on a body. The bodies themselves will no longer be considered to be perfectly rigid as was assumed in statics: instead the calculation of the deformations of various bodies under a variety of loads will be one of our primary concerns in the study of strength of materials.

The simplest case to consider at the start is that of an initially straight metal bar of constant cross section, loaded at its ends by a pair of oppositely directed collinear forces coinciding with the longitudinal axis of the bar and acting through the centroid of each cross section. For static equilibrium the magnitudes of the forces must be equal. If the forces are directed away from the bar, the bar is said to be in tension: if they are directed toward the bar, a state of compression exists. These two conditions are illustrated in Fig. 2.1.



a



b

Fig. 2.1

Under the action of this pair of applied forces internal resisting forces are set up within the bar .and their characteristics may be studied by imagining a plane to be passed through the bar anywhere along its length and oriented perpendicular to the longitudinal axis of the bar. Such a plane is designated as a-a in Fig. 2.1, a. If for purposes of analysis the portion of the bar to the right of this plane is considered to be removed. as in Fig. 2.1, b, then it must be replaced by whatever effect it exerts upon the left portion. By this technique of introducing a cutting plane the originally internal forces now become external with respect to the remaining portion of the body. For equilibrium of the portion to the left this "effect" must be a horizontal force of magnitude F. However, this force F acting normal to the cross-section is actually the resultant of distributed forces acting over this cross section in a direction normal to it.

At this point it is necessary to make some assumption regarding the manner of variation of these distributed forces and since the applied force F acts through the centroid it is commonly assumed that they are uniform across the cross section.

Instead of speaking of the internal force acting on some small element of area it is better for comparative purposes to treat the normal force acting over a unit area of the cross section. The intensity of normal force per unit area is termed the normal stress and is expressed in units of force per unit area, N/m$^{1.}$ lf the forces applied to the ends of the bar arc such that the bar is in tension, then tensile stresses are set up in the bar: if the bar is in compression we have compressive stresses. It is essential that the line of' action of the applied end forces pass through the centroid of each cross section of the bar.

The axial loading shown in Fig. 2.1 occurs frequently in structural and machine design problems. To simulate this loading in the laboratory, a test specimen is held in the grips of either an electrically driven gear-type testing machine or a hydraulic machine. Both of these machines are commonly used in materials testing laboratories for applying axial tension.

Let us suppose that one of these tension specimens has been placed in a tension-compression testing machine and tensile forces gradually applied to the ends. The elongation over the gage length may be measured as indicated above for any predetermined increments of the axial load. From these values the elongation per unit length, which is termed normal strain and denoted by ε, may be found by dividing the total elongation $\Delta\ell$ by the gage length ℓ. that is:

$$\varepsilon = \frac{\Delta\ell}{\ell}. \qquad (2.1)$$

The strain is usually expressed in units of inches per inch or meters per meter and consequently is dimensionless.

As the axial load is gradually increased in increments, the total elongation over the gage length is measured at each increment of load and this is continued until fracture of the specimen takes place. Knowing the original cross-sectional area of the test specimen the normal stress denoted by σ, may be obtained for any value of the axial load by the use of the relation:

$$\sigma = \frac{F}{A}, \qquad (2.2)$$

where F denotes the axial load in pounds or Newtons and A the original cross-sectional area. Having obtained numerous pairs of values of normal stress σ and normal strain ε, the experimental data may be plotted with these quantities considered as ordinate and abscissa respectively. This is the stress-strain curve or diagram of the material for this type of loading. Stress-strain diagrams assume widely differing forms for various materials. Figure 2.2 is the stress-strain diagram for a medium-carbon structural steel. Fig. 2.3 is for an alloy steel, and Fig. 2.4 is for hard steels and certain nonferrous alloys. For nonferrous alloys and cast iron the diagram has the form indicated in Fig. 2.5while for rubber the plot of Fig. 2.6 is typical.

Metallic engineering materials are commonly classed as either ductile or brittle materials. A ductile material is one having a relatively large tensile strain up to the point of rupture (for example structural steel or aluminum) whereas a brittle material has a relatively smal1 strain up to this same point. An arbitrary strain of 0.05 mm is frequently taken as the dividing line between these two classes of materials. Cast iron and concrete are examples of brittle materials.

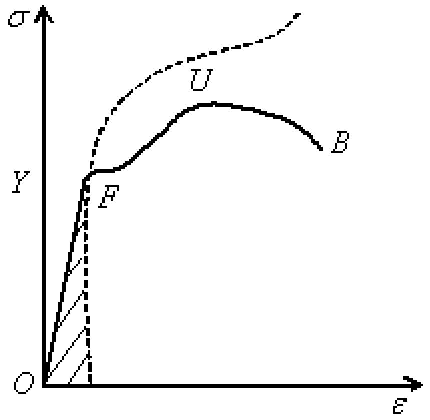


Fig. 2.2

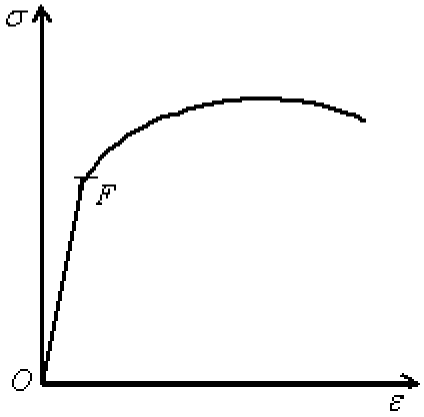


Fig. 2.3

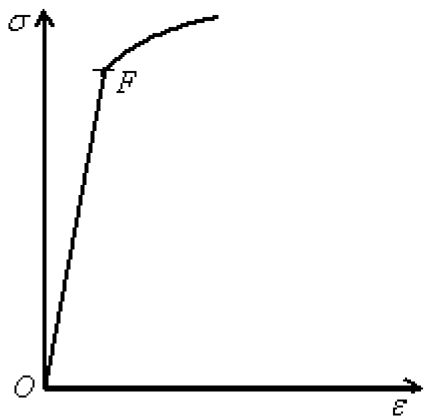


Fig. 2.4

For any material having a stress-strain curve of the form shown in Fig. 2.2, 2.3. or 2.4, it is evident that the relation between stress and strain is linear for comparatively small values of the strain. This linear relation between elongation and the axial force causing it (since these quantities respectively differ from the strain or the stress only by a constant factor) was first noticed by Sir Robert Hooke in 1678 and is called Hooke's law. To describe this initial linear range of action of the material we may consequently write:

$$\sigma = E\varepsilon , \tag{2.3}$$

where E denotes the slope of the straight - line portion OF of each of the curves in Figs. 2.2, 2.3 and 2.4.

The quantity E i.e. the ratio of the unit stress to the unit strain is the modulus of elasticity of the material in tension or as it is often

called, Youngs modulus. Values of E for various engineering materials are tabulated in handbooks. Since the unit strain E is a pure number (being a ratio of two lengths) it is evident that E has the same units as does the stress for example 1 N/m1. For many common engineering materials the modulus of elasticity in compression is very nearly equal to that found in tension. It is to be carefully noted that the behavior of materials under load as discussed in this book is restricted (unless otherwise stated) to the linear region of the stress -strain curve.
On beginning

2.2. Mechanical properties of materials

The stress-strain curve shown in Fig. 2.2 may be used to characterize several strength characteristics of the material. They are:

Propomonal Limit. The ordinate of the point F is known as the proporrional limir, i.e., the maximum stress that may be developed during a simple tension test such that the stress is a linear function of strain. For a material having the stress - strain curve shown in Fig. 2.5 there is no proportional limit.

Elastik Limit. The ordinate of a point almost coincident with F is known as the elastic limit, i.e., the maximum stress that may be developed during a simple tension test such that there is no permanent or residual deformation when the load is entirely removed. For many materials the numerical values of the elastic limit and the proportional limit are almost identical and the terms are sometimes used synonymously. In those cases where the distinction between the two values is evident the elastic limit is almost always greater than the proportional limit.

Elastic and Plastic Ranges. That region of the stress-strain curve extending from the origin to the proportional limit is called the elastic range, that region of the stress-strain curve extending from the proportional limit to the point of rupture is called the plastic range.

Yield Point. The ordinate of the point Y in Fig. 2.2 denoted by σ_{YF}, at which there is an increase in strain with no increase in stress is known as the yield point of the material. After loading has progressed to the point Y, yielding is said to take place. Some materials exhibit two points on the stress - strain curve at which there is an increase of strain without an increase of stress. These are called upper and lower yield points.

Ultimate Strength or Tensile Strength. The ordinate of the point U in Fig. 2.2, the maximum ordinate to the curve, is known either as the ultimate strength or the tensile strength of the material.

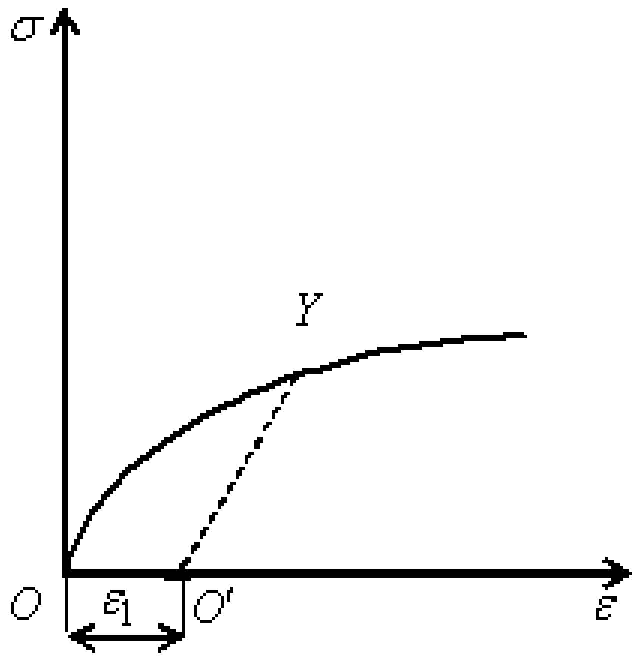


Fig. 2.5

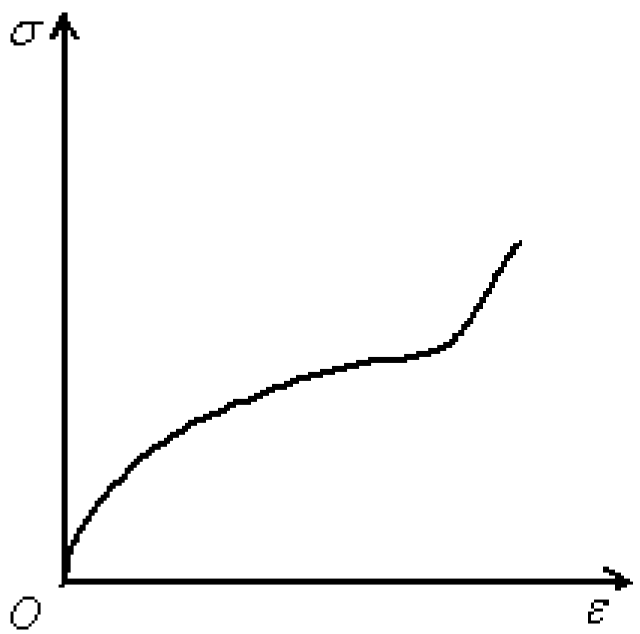


Fig. 2.6

Breaking Strength. The ordinate of the point B in Fig. 2.2 is called the breaking strength of the material.

Modulus of Resilience. The work done on a unit volume of material, as a simple tensile force is gradually increased from zero to such a value that the proportional limit of the material is reached, is defined as the modulus of resilience. This may be calculated as the area under the stress - strain curve from the origin up to the proportional limit and is represented as the shaded area in Fig. 2.2 The units of this quantity are in N·m/m3 in the SI system. Thus, resilience of a material is its ability to absorb energy in the elastic range.

Modulus of Toughness. The work done on a unit volume of material as a simple tensile force is gradually increased from zero to the value causing rupture is defined as the modulus of toughness. This may be calculated as the entire area under the stress-strain curve from the origin to rupture. Toughness of a material is its ability to absorb energy in the plastic range of the material.

Percentage Reduction in Area. The decrease in cross-sectional area from the original area upon fracture divided by the original area and multiplied by 100 is termed percentage reduction in area. It is to be noted that when tensile forces act upon a bar, the cross - sectional area decreases but calculations for the normal stress are usually made upon the basis of the original area. This is the case for the curve shown in Fig. 2.2 As the strains become increasingly larger it is more important to consider the instantaneous values of the cross - sectional area (which are decreasing).and if this is done the true stress - strain curve is obtained. Such a curve has the appearance shown by the dashed line in Fig. 2.1.

Percentage Elongation. The increase in length (of the gage length) after fracture divided by the initial length and multiplied by 100 is the percentage elongation. Both the percentage reduction in area and the percentage elongation are considered to be measures of the ductility of a material.

Working Sternness. The above - mentioned strength characteristics may be used to select a working stress. Frequently such a stress is determined merely by dividing either the stress at yield or the ultimate stress by a number termed the safety factor. SeLecture of the safety factor is based upon the designer's judgment and experience. Specific safety factors are sometimes specified in design codes.

Strain Hardening. If a ductile material can be stressed considerably beyond the yield point without failure it is said to strain - harden. This is true of many structural metals.

The nonlinear stress-s1rain curve of a brittle material shown in Fig. 2.5 characterizes several other strength measures that cannot be introduced if the stress-strain curve has a linear region. They are:

Yield Strength. The ordinate to the stress-strain curve such that the material has a predetermined permanent deformation or "set" when the load is removed is called the yield strength of the material. The permanent set is often taken to be either 0,002 or 0,0035 in mm. These values are of course arbitrary. In Fig. 2.5 a set ε_1 is denoted on the strain axis and the line $O'Y$ is drawn parallel to the initial tangent to the curve. The ordinate of Y represents the yield strength of the material sometimes called the proof stress.

Tangent Modulus. The rate of change of stress·with respect to strain is known as the tangent modulus of the material. It is essentially an instantaneous modulus given:

$$E_1 = \frac{d\sigma}{d\varepsilon}.$$

Coefficient of Linear Expansion. This is defined as the change of length per unit length of a straight bar subject to a temperature change of one degree and is usually denoted by α. The value of this coefficient is independent of the unit of length but does depend upon the temperature scale used. Temperature changes in a structure give rise to internal stresses just as do applied loads.

Poisson's Ratio. When a bar is subject to a simple tensile loading there is an increase in length of the bar in the direction of the load but a decrease in the lateral dimensions perpendicular to the load. The ratio of the strain in the lateral direction to that in the axial direction is defined as Poisson's ratio. It is denoted in this book by the Greek letter μ. For most metals it lies in the range 0,25 to 0,35. For cork, μ is very nearly zero. One new and unique material so far of interest only in laboratory investigations actually has a negative value of Poisson's ratio, i.e., if stretched in one direction it expands in every other direction.

On beginning

2.3. General form of Hooke's law

The simple form of Hooke's law has been given for axial tension when the loading is entirely along one straight line i.e., uniaxial. Only the deformation in the direction of the load was considered and it was given by:

$$\varepsilon = \frac{\sigma}{E}.$$

In the more general case an element of material is subject to three mutually perpendicular normal stresses σ_x. σ_y, σ_z, which are accompanied by the strains ε_x ε_y, ε_z, respectively. By superposing the strain components arising from lateral contraction due to Poisson's effect upon the direct strains we obtain the general s1atemen1 of Hooke's law:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \mu\left(\sigma_y + \sigma_z\right)\right];$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \mu\left(\sigma_z + \sigma_x\right)\right]; \qquad (2.4)$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \mu\left(\sigma_z + \sigma_x\right)\right].$$

Specifk Strength. This quantity is defined as the ratio of the ultimate (or tensile) strength to specific weight. i.e., weight per unit volume. Thus, in the SI system.we have H/m^2 = H/m^3=m, so that in either system specific strength has units of length. This parameter is useful for comparisons of material efficiencies.

Specifk Modulus. This quantity is defined. as the ratio of the Young's modulus to specific weight. Substitution of units indicates that specific modulus has physical units of length in SI systems.

In determination of mechanical properties of a material through a tension or compression test the rate at which loading is applied sometimes has a significant influence upon the results. In general, ductile materials exhibit the greatest sensitivity to variations in loading rate, whereas the effect of testing speed on brittle materials, such as cast iron has been found to be negligible. In the case of mild steel, a ductile material, it has been found that the yield point may be increased as much as 170 percent by extremely rapid application of axial force. It is of interest 10 note, however, that for this case the total elongation remains unchanged from that found for slower loadings.

Stresses and deformations in the plastic range of action of a material are frequently permitted in certain structures. Some building codes allow particular structural members to undergo plastic deformation, and certain components of aircraft and missile structures are deliberately designed to act in the plastic range so as to achieve weight savings.

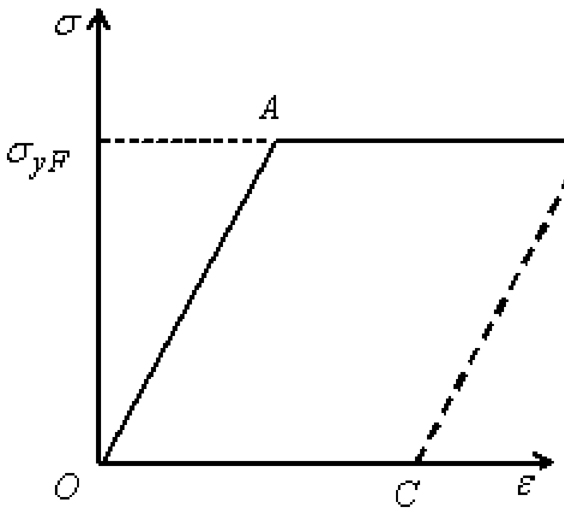


Fig. 2.7

Furthermore, many metal-forming processes involve plastic action of the material. For small plastic strains of low- and medium-carbon structural steels the stress-strain curve of Fig. 2.7 is usually idealized by two straight lines, one with a slope of E, representing the elastic range, the other with zero slope representing the plastic range. This plot, shown in Fig. 2.7, represents a so-called elastic, perfectly plastic material. It takes no account of still larger plastic strains occurring in the strain-hardening region shown as the right portion of the stress-strain curve of Fig. 2.1.

If the load increases so as to bring about the strain corresponding to point B in Fig. 2.7 and then the load is removed, unloading takes place along the line BC so that complete removal of the load leaves a permanent "set" or elongation corresponding to the strain OC.

We shall consider typical example.

Example 1.1.

The pinned members shown in Fig. 2.8 carry the loads F and $2F$. All bars have cross-sectional area A.

Determine the stresses in bars AB and AK.

The reactions are indicated by R_{Cx}, R_{Cy} and R_A. From statics we have:

$$\sum M_{C_i} = 0, \qquad \text{or} \qquad -2F \cdot \ell - F \cdot 2\ell + R_A \cdot 3\ell = 0,$$

$$R_A = \frac{4}{3}F.$$

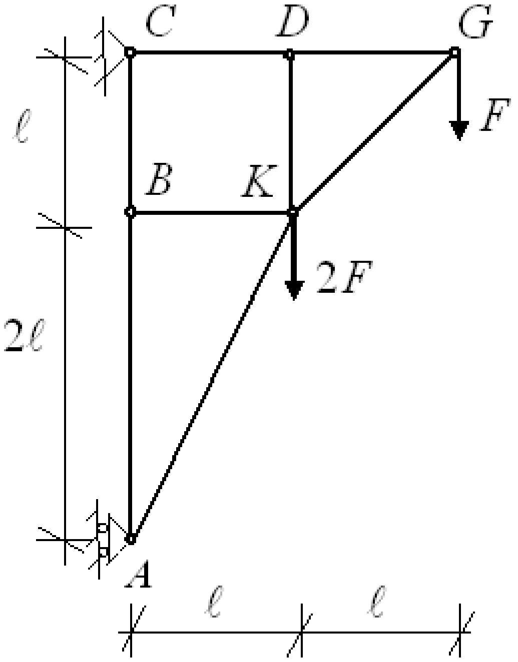


Fig. 2.8

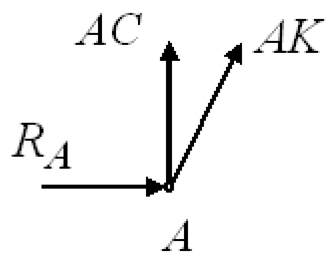


Fig. 2.9

A free-body diagram of the pin at A is shown in Fig. 2.9. From statics:

$$\sum F_{x_i} = 0, \text{or} \quad \frac{4}{3}F + \frac{1}{\sqrt{5}}AK = 0, \quad AK = -\frac{4\sqrt{5}}{3}F;$$

$$\sum F_{y_i} = 0, \text{or} \quad AB + \frac{2}{\sqrt{5}}AK = 0, \quad AB = -\frac{8}{3}F.$$

The bar stresses are:

$$\sigma_{AK} = -\frac{4\sqrt{5}}{3A}F; \qquad \sigma_{AB} = -\frac{8F}{3A}.$$

On beginning