Lecture 5
THE GEOMETRIC CHARACTERIZTIONS OF THE PLANE CROSS SECTIONS

Plan

1. First moment of area.
2. Centroid of an area.
3. Moments of inertia.

5.1. First moment of area.

The first moment of an element of area about any axis in the plane of the area is given by the product of the area of the element and the perpendicular distance between the element and the axis.

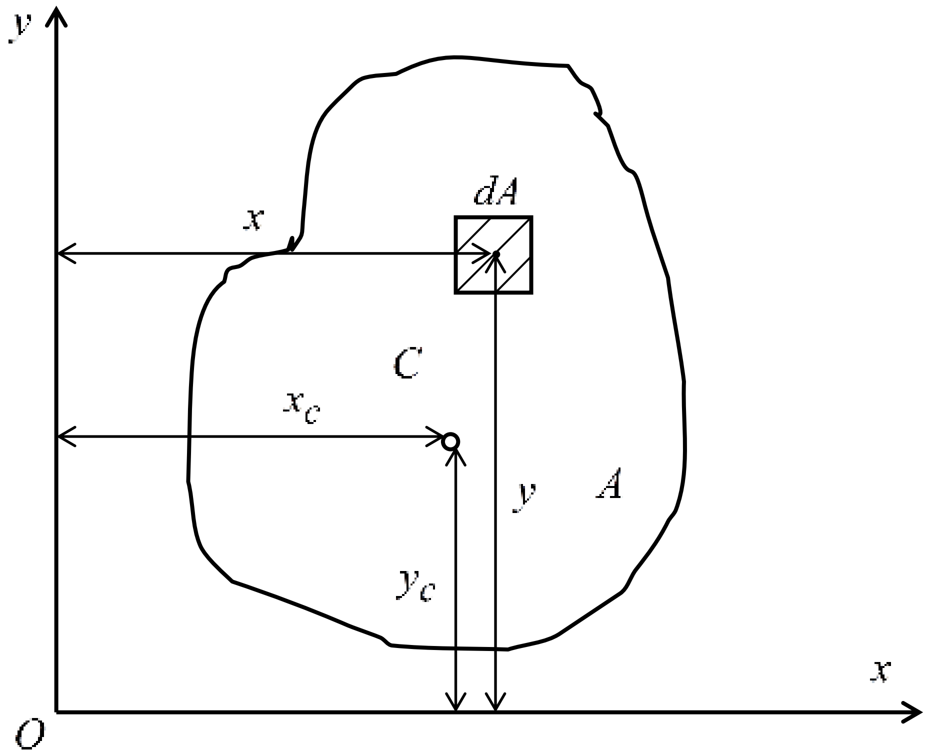

Fig. 5.1

For example, in Fig. 5.1 the first moment dS, of the element dA about the x-axis is given by:

$$dS_x = ydA.$$

About the y-axis the first moment is:

$$dS_y = xdA.$$

Let us consider some examples on this topic.

Example 1.

Define the static moments of parts of area a section beams which are located higher line 2-2 in relation to central main axis x (Fig. 5.2).

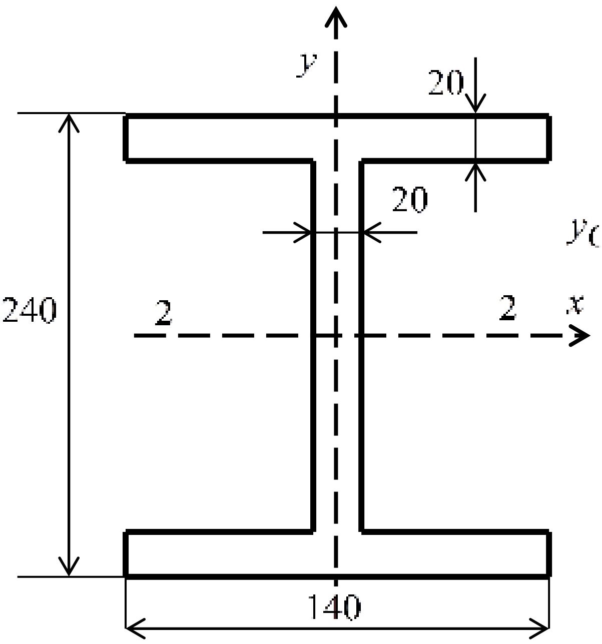

Fig. 5.2

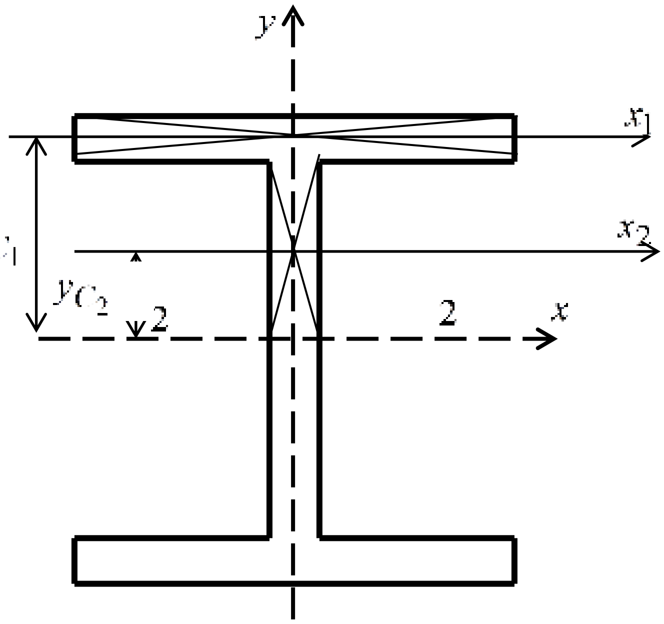

Fig.5.3

Breaking up this section on three rectangles with according areas:

$$A_1 = 2 \times 14 = 28 \text{ cm}^2, \quad A_2 = 2 \times 10 = 20 \text{ cm}^{5.}$$

Conduct through a center scales each of rectangles own central axes, accordingly x_1, x_2 (Fig. 5.3).

Then the static moment of area in relation to an axis which coincides straight-in 2 - 2 will be written down so:

$$S_x^{2-2} = A_1 \cdot y_{C_1} + A_2 y_{C_2} = 28 \cdot 11 + 20 \cdot 5 = 408 \text{ cm}^{5.}$$

Example 2.

Determine the static moment of area of a section in relation to an axis x_0 (Fig. 5.4).

The figure consists of two rectangles, one of them is openings, that is a fictitious area.

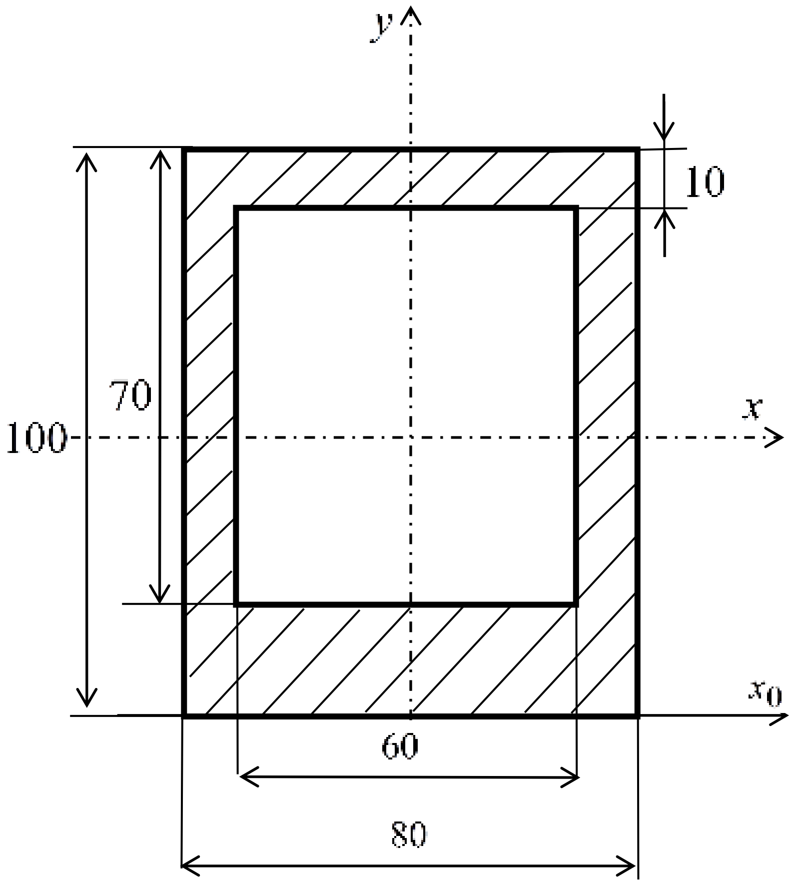

Fig. 5.4

Calculate an area each of rectangles:

$$A_1 = 10 \cdot 8 = 80 \text{ cm}^2; \quad A_2 = 7 \cdot 6 = 42 \text{ cm}^{5.}$$

Axis x - is the central axis of the first rectangle and it coincides with an axis x_1, conduct a central axis for opening x_2 (Fig. 5.5). Then distances from axes x_1 and x_2 accordingly will be equal:

$$y_{C_1} = 5 \text{ cm}; \qquad y_{C_2} = 3 + 3{,}5 = 6{,}5 \text{ cm}.$$

Let us calculate the first moments of each of rectangles, we get:

$$S_{x_{10}} = A_1 \cdot y_{C_1} = 80 \cdot 5 = 400 \text{ cm}^3;$$

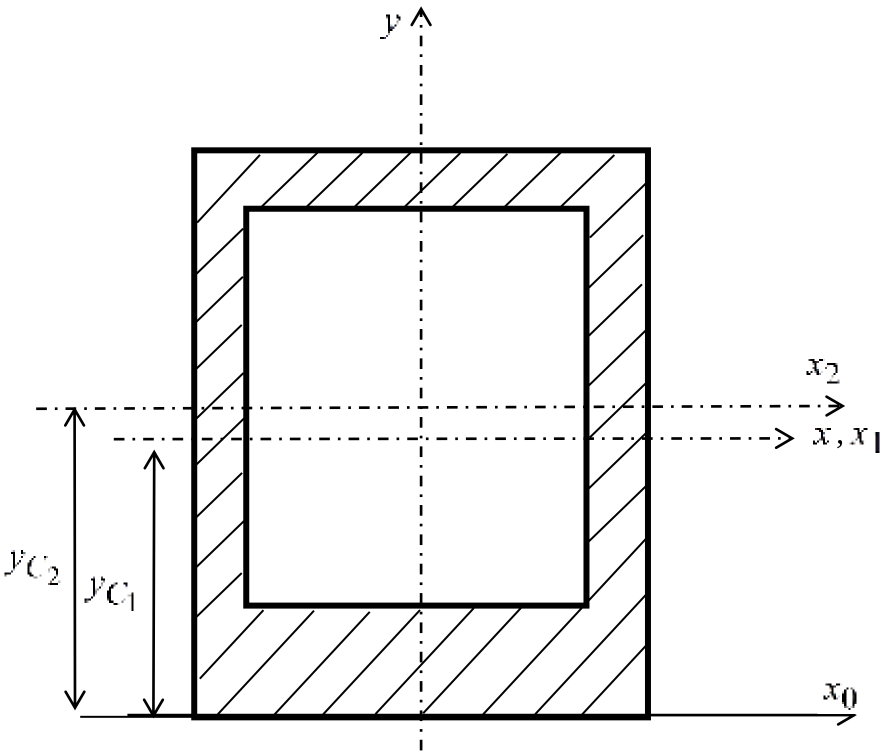

Fig. 5.5

$$S_{x_{20}} = A_2 \cdot y_{C_2} = 42 \cdot 6{,}5 = 273 \text{ cm}^{3.}$$

Than:

$$S_{x_0} = S_{x_{10}} - S_{x_{20}} = 400 - 273 = 127 \text{ cm}^{3.}.$$

Consequently, as evidently from this example, although the sign of sum stands, but in the case when opening comes forward at a role component a section, his area is considered fictitious, that such which must be taken with a sign „minus”. And that is why static moments in this case were not added, but subtracted.

The first moment of a finite area about any axis in the plane of the area is given by the summation of the first moments about that same axis of all the elements of area contained in the finite area. This is frequently evaluated by means of an integral. If the first moment of the finite area is denoted by S_x then: $S_x = \int dS_x$. **On beginning**

5.2. Centroid of an area.

The centroid of an area is defined by the equations:

$$y_C = \frac{S_x}{A} = \frac{\int y dA}{A}, \qquad x_C = \frac{S_y}{A} = \frac{\int x dA}{A}, \tag{5.1}$$

where A denotes the area.

For a plane area, which is composed of n subareas A_i, each of whose centroidal coordinates x_C and y_C are known, the integral is replaced by a summation:

$$x_C = \frac{S_y}{A} = \frac{\sum\limits_{i=1}^{n} A_i x_{C_i}}{\sum\limits_{i=1}^{n} A_i}, \qquad y_C = \frac{S_x}{A} = \frac{\sum\limits_{i=1}^{n} A_i y_{C_i}}{\sum\limits_{i=1}^{n} A_i}. \tag{5.2}$$

The centroid of an area is the point at which the area might be considered to be concentrated and still leaves unchanged the first moment of the area about any axis. For example, a thin metal plate will balance in a horizontal plane if it is supported at a point directly under its center of gravity.

The centroids of a few areas are obvious. In a symmetrical figure such as a circle or square, the centroid coincides with the geometric center of the figure.

It is common practice to denote a centroid distance by a bar over the coordinate distance. Thus, x indicates the x - coordinate of the centroid.

Example 3.

On Fig. 5.6 asymmetrical transversal a section is given.

Determine position of centeroid of cross-section and conduct central axes x_C and y_C.

Geometrical sizes of section to take from fig.5.6.

For given section conduct initial auxiliary axes x_0 and y_0. They are passes along generator given of the section according to Fig. 5.7.

Area of given section it is possible to give as a sum of areas of three rectangles, and it is possible to give and as a difference two.

I method. At first will define position of center of weight, taking for basis the first variant of split of given section into constituents. Then by Fig. 5.7 areas of the corresponding rectangles there will be equals:

$$A_1 = 2 \times 6 = 12 \text{ cm}^2; \quad A_2 = 3{,}5 \times 6 = 21 \text{ cm}^2;$$

$$A_3 = 9{,}5 \times 4 = 38 \text{ cm}^{5.}$$

Conduct central axes for each of three rectangles and define distances from them y_{C_i} and x_{C_i} to the initial auxiliary axes x_0 and y_0:

1. $x_{C_1} = \frac{2}{2} = 1$ cm; $\quad y_{C_1} = 3 + 4 = 7$ cm; $\quad A_1 = 12$ cm^2;

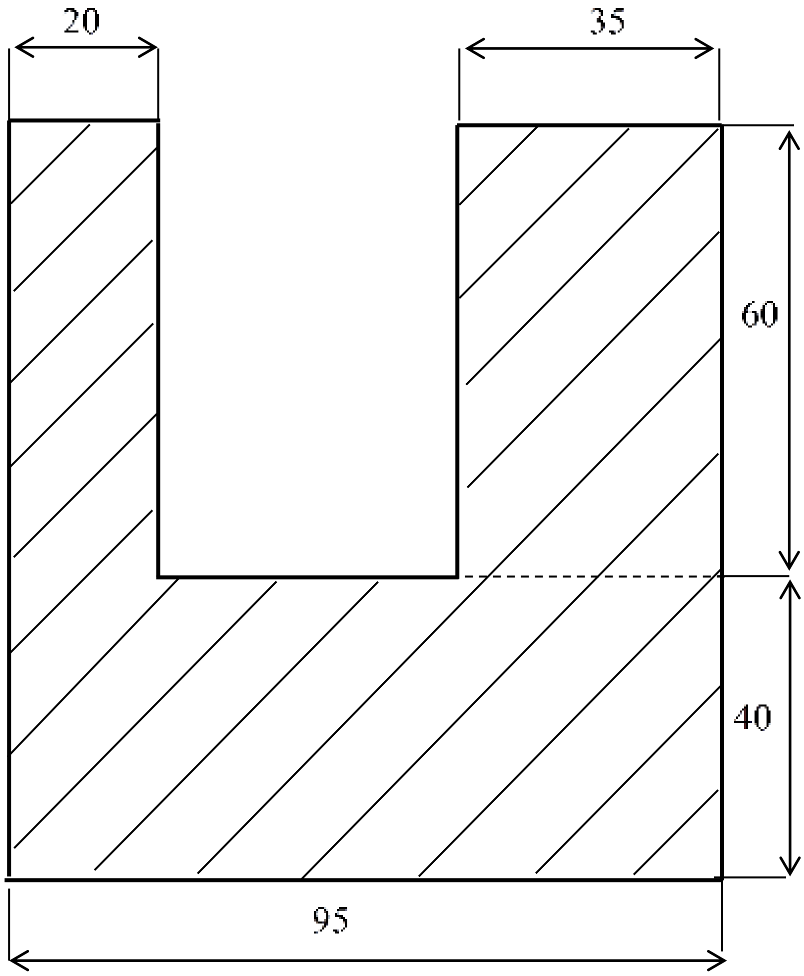

Fig. 5.6

5. $x_{C_2} = 9{,}5 - \frac{3{,}5}{2} = 7{,}75\ \text{cm};\qquad y_{C_2} = 3 + 4 = 7\ \text{cm};\qquad A_2 = 21\ \text{cm}^2;$

5. $x_{C_3} = \frac{9{,}5}{2} = 4{,}75\ \text{cm};\qquad y_{C_3} = \frac{4}{2} = 2\ \text{cm};\qquad A_3 = 38\ \text{cm}^{5.}$

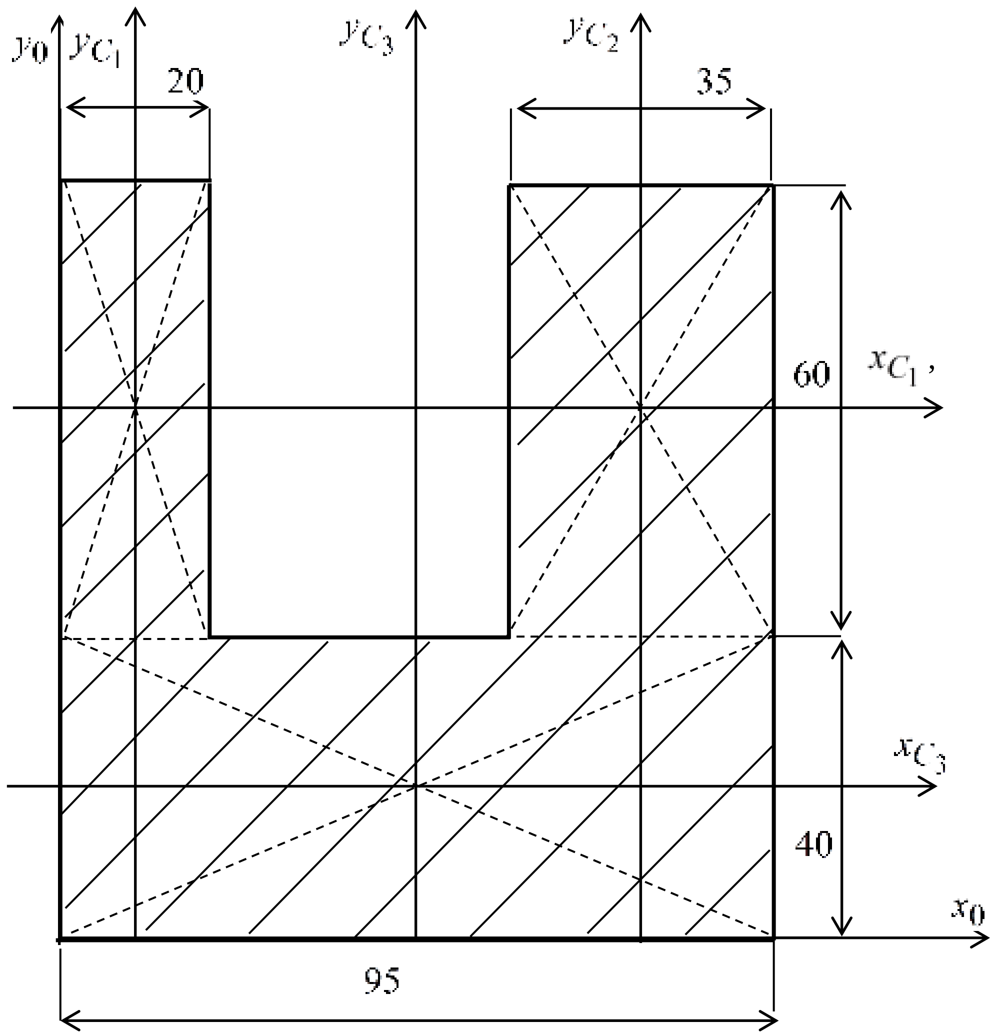

Fig. 5.7

Then in obedience to formula (5.2), we get:

$$x_C = \frac{S_y}{A} = \frac{\sum_{i=1}^{n} A_i x_{C_i}}{\sum_{i=1}^{n} A_i} = \frac{x_{C_1} \cdot A_1 + x_{C_2} \cdot A_2 + x_{C_3} \cdot A_3}{A_1 + A_2 + A_3} =$$

$$= \frac{1 \cdot 12 + 7{,}75 \cdot 21 + 4{,}75 \cdot 38}{12 + 21 + 38} = \frac{355{,}25}{71} \approx 5 \text{ cm;}$$

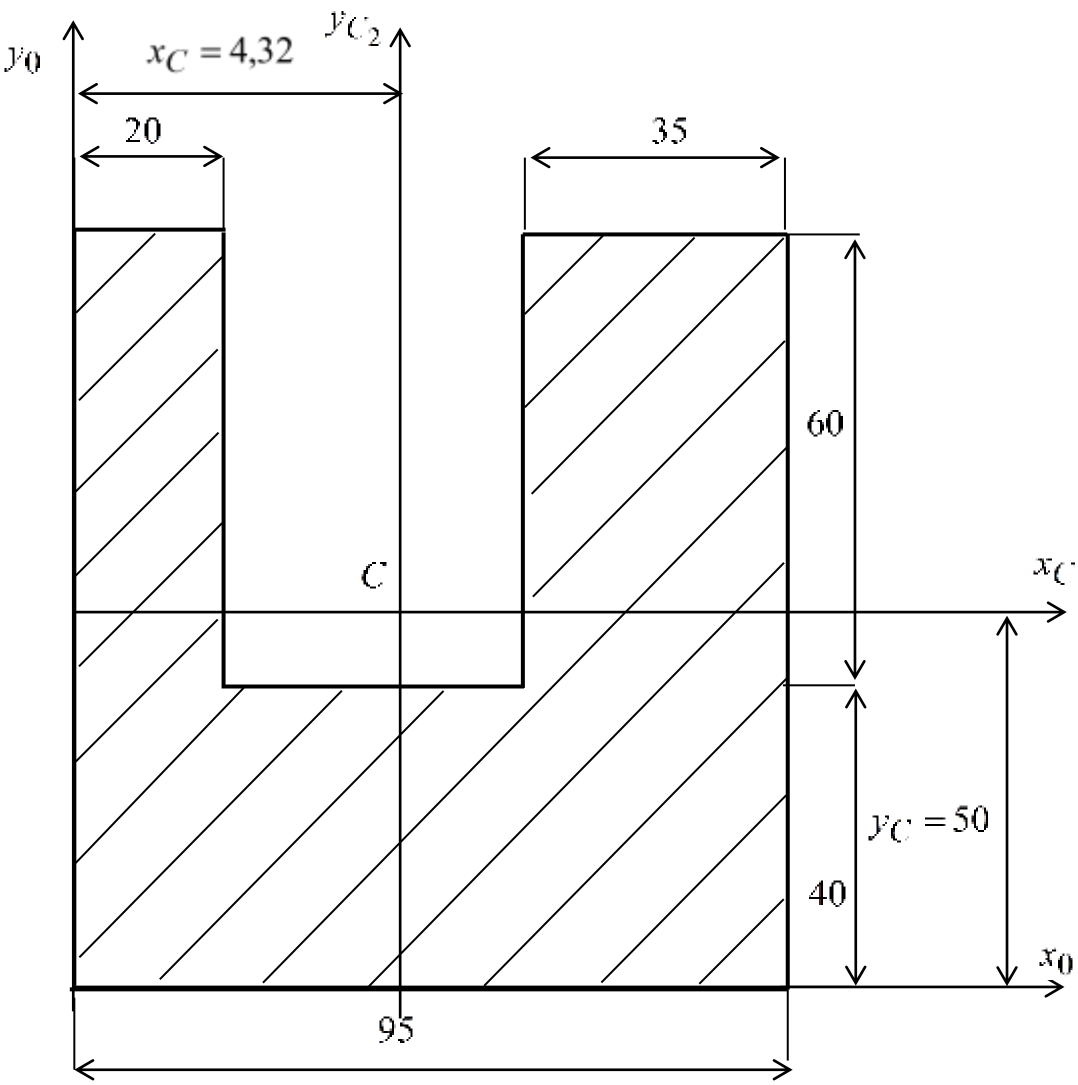

Fig. 5.8

$$y_C = \frac{S_x}{A} = \frac{\sum_{i=1}^{n} A_i y_{C_i}}{\sum_{i=1}^{n} A_i} = \frac{y_{C_1} \cdot A_1 + y_{C_2} \cdot A_2 + y_{C_3} \cdot A_3}{A_1 + A_2 + A_3} =$$

$$= \frac{7 \cdot 12 + 7 \cdot 21 + 2 \cdot 38}{12 + 21 + 38} = \frac{307}{71} \approx 4{,}32 \text{ cm.}$$

Thus, corresponding to the obtained calculations, in order that an initial auxiliary axis x_0 became central for given section, it needs to be lifted up on a size 4,32 cm, accordingly axis y_0 - to displace to the right on 5 sm. In the total we get the new pair of axes x_C and y_C and, which for given section is central axes. Their intersection is the center of weight for given section – point C (Fig. 5.8).

II methord. Let us consider other variant of dividing a given cross section. Namely, it consists of large rectangle, which has the width $b_1 = 9{,}5$ cm and the height $h_1 = 10$ cm and the rectangular opening, which has width $b_2 = 4$ cm, and height of $h_2 = 6$ cm (Fig. 5.9).

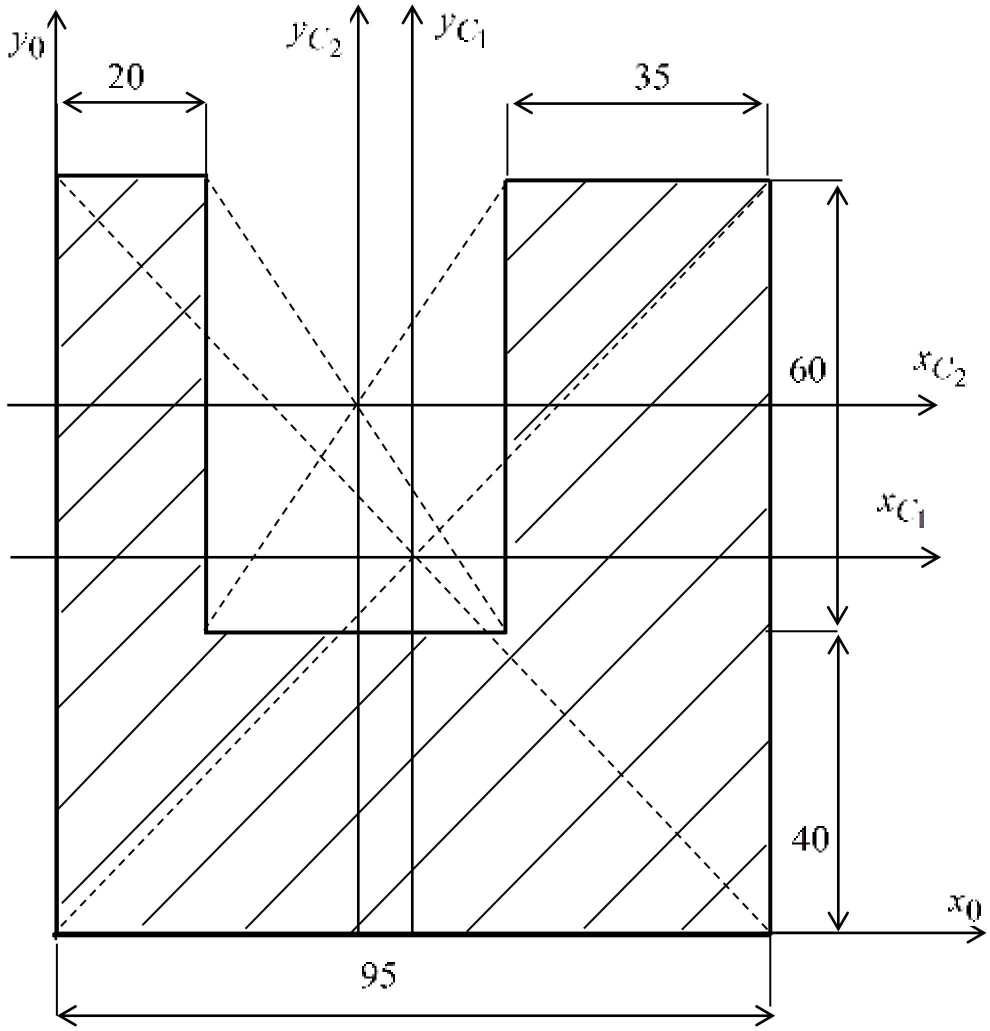

Fig. 5.9

Then:

$$A_1 = 9{,}5 \times 10 = 95 \text{ cm}^2; \qquad A_2 = 4 \times 6 = 24 \text{ cm}^{5.}$$

Let us conduct central axes for each of two rectangles and determine distances from them y_{C_i} and x_{C_i} to the initial auxiliary axes x_0 and y_0:

1. $x_{C_1} = \frac{9{,}5}{2} = 4{,}75$ cm; $y_{C_1} = \frac{10}{2} = 5$ cm; $A_1 = 95$ cm^2;

5. $x_{C_2} = 2 + \frac{4}{2} = 4$ cm; $y_{C_2} = 4 + \frac{6}{2} = 7$ cm; $A_2 = 24$ cm$^{5.}$

In this case formulas (5.2) acquire a kind:

$$x_C = \frac{S_y}{A} = \frac{x_{C_1} \cdot A_1 - x_{C_2} \cdot A_2}{A_1 - A_2} = \frac{4{,}75 \cdot 95 - 4 \cdot 24}{95 - 24} = \frac{355{,}25}{71} \approx 5 \text{ cm};$$

$$y_C = \frac{S_x}{A} = \frac{y_{C_1} \cdot A_1 - y_{C_2} \cdot A_2}{A_1 - A_2} = \frac{5 \cdot 95 - 7 \cdot 24}{95 - 24} = \frac{307}{71} \approx 4{,}32 \text{ cm}.$$

Consequently, as see, we got identical results by second method, that and first, but the calculations are the far fewer. And it means that the second method in comparing to the first is more rational. Because in time on the same job problem needs far fewer. It is in addition, important to understand that in a formula (5.2) sign a symbolic value has a plus how in the case of presence in the cut of opening it transforms in relation to the last on a sign minus.

On beginning

5.3. Moments of inertia

The second moment or moment of inertia of an element of area about any axis in the plane of the area is given by the product of the area of the element and the square of the perpendicular distance between the element and the axis. In Fig. 5.l. the moment of inertia dI_x, of the element about the x - axis is:

$$dI_x = y^2 dA.$$

About the y - axis the moment of inertia is:

$$dI_y = x^2 dA.$$

The second moment or moment of inertia of a finite area about any axis in the plane of the area is given by the summation of the moments of inertia about that same axis of all of the elements of area contained in the finite area. This, too, is frequently found by means of an integra1. If the moment of inertia of the finite area about the x - axis is denoted by I_x, then we have:

$$I_x = \int dI_x = \int y^2 dA. \tag{5.3}$$

Example 4.

Determine the moment of inertia of a rectangle about an axis through the centroid and parallel to the base.

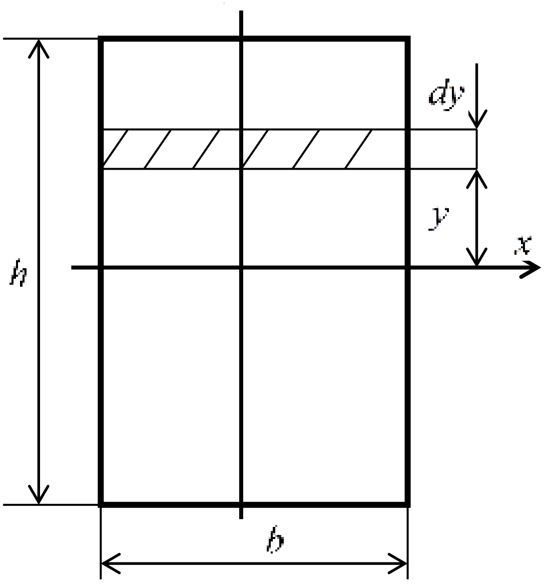

Fig. 5.10

Let us introduce the coordinate system shown in Fig. 5.10. The moment of inertia I_x about the x - axis passing through the centroid is given by (5.3). For convenience it is logical to select an element such that y is constant for all points in the element. The shaded area shown has this characteristic.

$$I_x = \int_A y^2 dA = \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 bydy = \int_{-\frac{h}{2}}^{\frac{h}{2}} y^3 bdy = 2b\int_0^{\frac{h}{2}} y^3 dy = \frac{bh^3}{12}. \tag{5.4}$$

On beginning