# Lecture 10
# THE CALCULATION OF THE BAR ON STRENGTH AND RIGIDITY BY TORSION

Plan

1. The determination of the inside and outside diameter of the shaft from on rigidity by torsion.
2. Typical example of calculation of shaft by torsion.
3. The statically indeterminate problem under torsion.

**10.1**. **The determination of the inside and outside diameter of the shaft from on rigidity by torsion**.

A hollow steel shah 3 m long must transmit a torque or 25 kNm. The total angle of twist in this length is not to exceed $2{,}5^0$ and the allowable shearing stress is 90 MPa.

Determine the inside and outside diameter of the shaft if $G = 85$ GPa.

Let $D$ and $d$, designate the outside and inside diameters of the shaft, respectively. From Eq.(9.2) the angle of twist is $\theta = {T\ell}/{GI_\rho}$, where $\theta$ is expressed in radians. Thus, in the 3-m length we have:

$$\frac{2{,}5}{57{,}3} = \frac{25000 \cdot 3}{85 \cdot 10^9 \frac{\pi}{32}\left(D^4 - d^4\right)},$$

or

$$D^4 - d^4 = 206 \cdot 10^{-6} \text{ m}^{3.}$$

The maximum shearing stress occurs at the outer fibers where $\rho = \dfrac{D}{2}$. At these points from Eq. (9.2), we get:

$$90 \cdot 10^6 = \frac{25000 \cdot D/2}{\frac{\pi}{32}\left(D^4 - d^4\right)},$$

or

$$D^4 - d^4 = 1414 \cdot D \cdot 10^{-6} \text{ m}^{3.}$$

Comparison of the right-hand sides of these equations indicates that:

$$206 \cdot 10^{-6} = 1414 \cdot D \cdot 10^{-6}$$

and thus $D = 0{,}145$ m or 145 mm. Substitution of this value into either of the equations then gives $d = 0{,}125$ m or 125 mm.

**On beginning**

**10.2. Typical example of calculation of shaft by torsion.**

Let for steel shaft (see Fig.10.1, a) is set: $a = b = c = 1{,}5$ m, $[\tau] = 55$ kPa, $M_1 = M_2 = M_3 = M_4 = 150$ Nm.

It is necessary to draw the diagrams of torques and angle of twist, and also from the condition of durability to pick up the round transversal section of shaft

1. Drawing the diagram of torques $T$ (Fig. 10.1, b):

I portion. $0 \le x_1 \le 1{,}5$ m. $T_1 = M_4 = 150$ Nm;

II portion. $1{,}5 \text{ m} \le x_2 \le 3$ m. $T_2 = M_4 - M_3 = 0$;

III portion. $3 \text{ м} \le x_3 \le 4{,}5$ m. $T_3 = M_4 - M_3 - M_2 = -150$ Nm;

IV portion. $4{,}5 \text{ m} \le x_4 \le 6$ m. $T_4 = M_4 - M_3 - M_2 + M_1 = 0$.

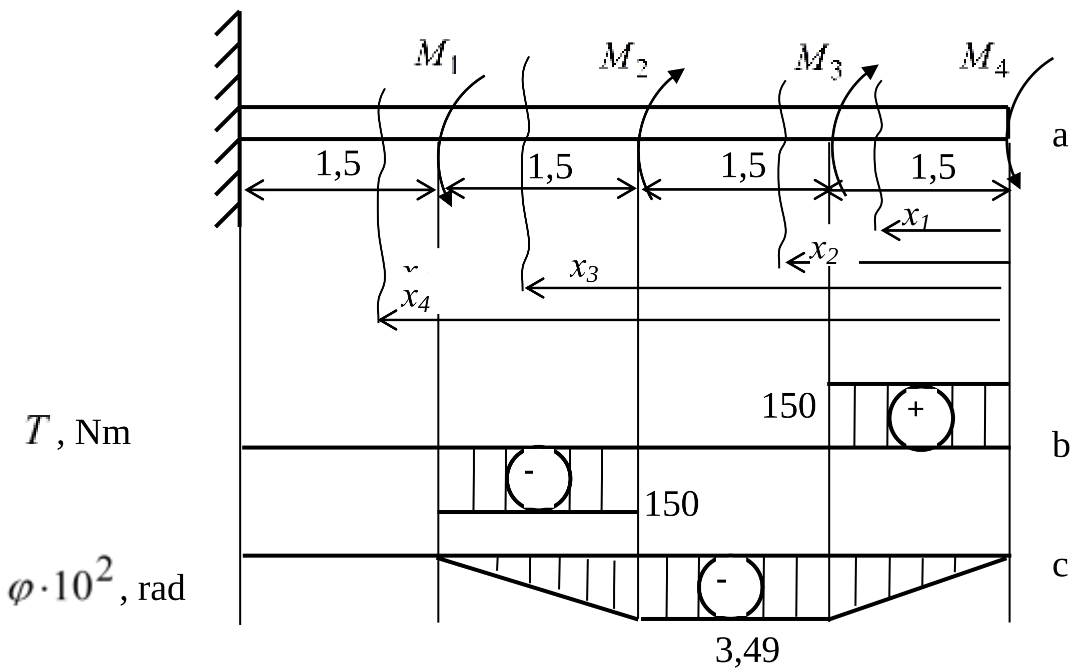


Fig. 10.1

2. From the condition of durability the diameter of shaft is determine:

$$d \geq \sqrt[3]{\frac{16 \cdot T_{max}}{\pi \cdot [\tau]}},$$

in this case we have $T_{max} = 150$ Nm.

Then:

$$d \geq \sqrt[3]{\frac{16 \cdot 150}{3{,}14 \cdot 55 \cdot 10^3}} \approx 0{,}024 \text{ m} = 24 \text{ mm}.$$

Thus, $d = 30$ mm.

Then:

$$I_\rho = \frac{\pi d^4}{32} = \frac{3{,}14 \cdot 3^4}{32} \approx 7{,}95 \text{ cm}^4 = 7{,}95 \cdot 10^{-8} \text{ m}^4.$$

3. Taking into account, that for steel $G = 8{,}1 \cdot 10^{10}$ Pa, let us calculate the angle of twist on every area and draw the diagram of angle of twist (Fig. 10.1, c)

I portion. $0 \leq x_1 \leq 1{,}5$ m. $\qquad \varphi_1 = 0$;

II portion. $1{,}5 \text{ m} \leq x_2 \leq 3$ m.

$$\varphi_2 = \frac{-150 \cdot 1{,}5}{8{,}1 \cdot 10^{10} \cdot 7{,}95 \cdot 10^{-8}} \approx 3{,}49 \cdot 10^{-2} \text{ rad};$$

III portion. $3 \text{ m} \leq x_3 \leq 4{,}5$ m. $\qquad \varphi_3 = \varphi_2 = 3{,}49 \cdot 10^{-2}$ rad;

IV portion. $4{,}5 \text{ m} \leq x_4 \leq 6$ m.

$$\varphi_4 = \varphi_3 + \frac{150 \cdot 1{,}5}{8{,}1 \cdot 10^{10} \cdot 7{,}95 \cdot 10^{-8}} = 0 \text{ rad}.$$

**On beginning**

**10.3. The statically indeterminate problem under torsion.**

Let us determine the reactive torques at the fixed ends of the circular shaft loaded by the couples shown in Fig. 10.2. The cross section of the bar is constant along the length. Assume elastic action.

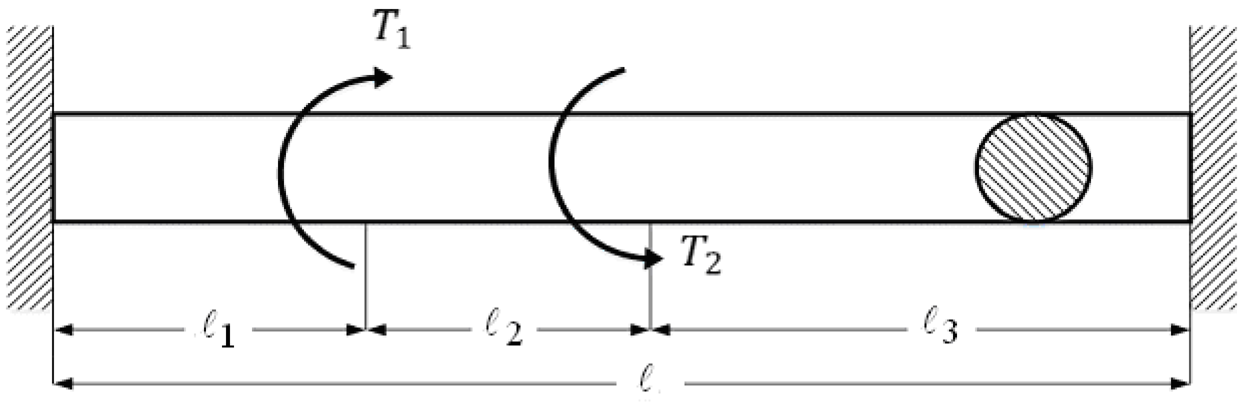


Fig. 10.2

Let us assume that the reactive torques $T_A$ and $T_B$ are positive in the directions shown in Fig. 10.3.

From static we have:

$$T_A - T_1 + T_2 - T_B = 0. \qquad (10.1)$$

This is the only equation of static equilibrium and it contains two unknowns. Hence this problem is statically indeterminate and it is necessary to augment this equation with another equation based on the deformations of the system.

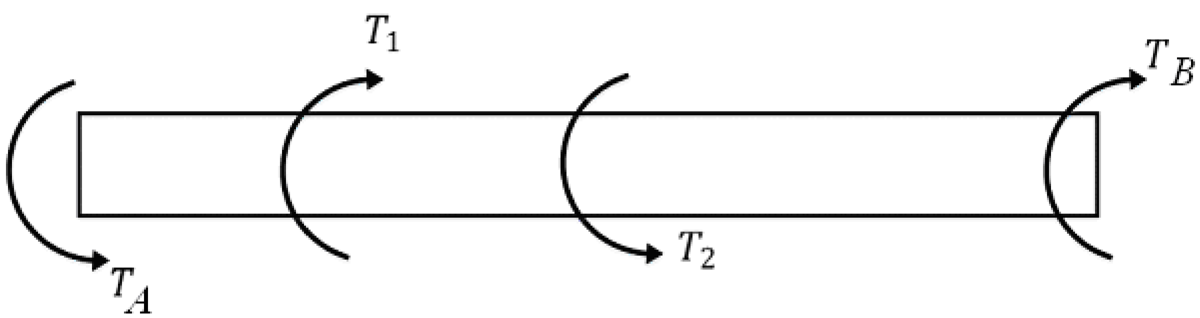


Fig. 10.3

The free - body diagram of the left region of length $L_1$ appears as in Fig. 10.4, a.

Working from left to right along the shaft, the twisting moment in the central region of length $L_2$ is given by the algebraic sum of the torques to the left of this section, i.e., $T_1$ - $T_A$. The free - body diagram of this region appears as in Fig. 10.4. Finally, the free-body diagram of the right region of length $\ell$ appears as in Fig. 10.4.

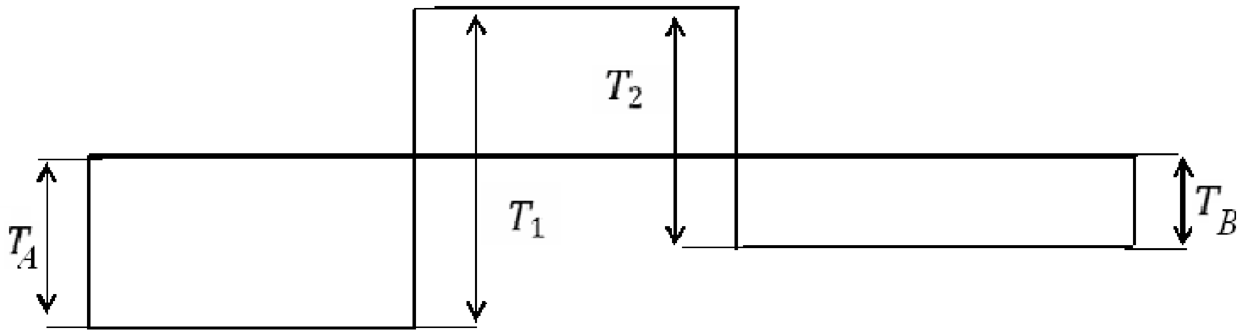


Fig. 10.4

Let $\theta_1$ denote the angle of twist at the point of application of $T_1$ and $\theta_2$ the angle at $T_2$. Then from a consideration of the regions of lengths $\ell_1$ and $\ell_3$ we immediately have:

$$\theta_1 = \frac{T_A \ell_1}{GI_\rho}; \qquad \theta_2 = \frac{T_B \ell_3}{GI_\rho}. \tag{10.2}$$

The original position of a generator on the surface of the shaft is shown by a solid line in Fig. 10.4, and the deformed position by a dashed line. Consideration of the central region of length $\ell_2$ reveals that the angle of twist of its right end with respect to its left end is $\theta_1 + \theta_2$. Hence, since the torque causing this deformation is $T_1 - T_A$, we have:

$$\theta_1 + \theta_2 = \frac{(T_1 - T_A)\ell_2}{GI_\rho}. \tag{10.3}$$

Solving (3.8) through (3.10) simultaneously, we find:

$$T_A = T_1 \frac{\ell_2 + \ell_3}{\ell} \quad \text{and} \quad T_B = -T_1 \frac{\ell_1}{\ell} + T_2 \frac{\ell_1 + \ell_2}{\ell}.$$

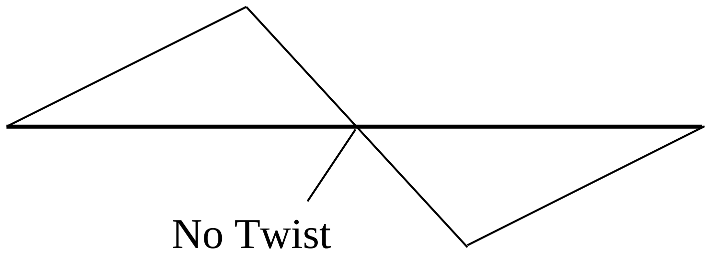


Fig. 10.5

It is of interest to examine the behaviour of a generator on the surface of the shaft. Originally it was. of course, straight over the entire length $\ell$, but after application of $T_1$ and $T_2$ it has the appearance shown by the broken line in Fig. 10.5.

**On beginning**