Lecture 17
VERESCAGIN'S RULE

Plan

1. General theory.
2. The first example of application.
3. The second example of application.

17.1. General theory. This method is one of popular methods of determination of deformations of beams and frames. Essence of this method consists, in multiplying of diagrams of beadings moments by a formula:

$$\delta = \sum_{i=1}^{n} \frac{\omega_i \cdot M_{C_i}}{EI}, \tag{17.1}$$

where ω_i represents the elementary moment area; M_{C_i} is the ordinate of the unite force moment under the centroid of ω_i.

In according to this method consider the load state and the unit state of beam. The load state is corresponds the case when the beam is loading by only the given external loading. The unit state of beam is corresponding to the case, when a beam is loaded with only unity effort. It may be the unity bending moment or unity concentrated force.

If in obedience to a problem specification it is necessary to define bending of beam or frame, in a point where it needs to be defined, load the unit concentrated force $F = 1$. If it is necessary to find the slope of the deflection curve, put a unit bending moment $M = 1$.

Two diagrams of bending moment for the load state M^F and for the unit state M^1 are building, which are used in a formula (17.1).

On beginning

17.2. The first example of application.

Determine by Verescagin's rule the deflection at point B of the cantilever beam subject to the single concentrated force P, as shown in Fig. 17.1, a.

Let us build two epures of bending moment for the load state M^F (Fig. 17.1, b) and for the unit state M^1 (Fig. 17.1, c).

From Fig. 17.1, b we have:

$$\omega = \frac{1}{2}(-PL)\cdot L = -\frac{PL^2}{2}.$$

From Fig. 17.1, c we find:

$$M_C = -\frac{2}{3}L.$$

By a formula (17.1) we obtain:

$$\delta_B = \frac{1}{EI_x}\left(-\frac{PL^2}{2}\right)\left(-\frac{2}{3}L\right) = \frac{PL^3}{3EI_x}.$$

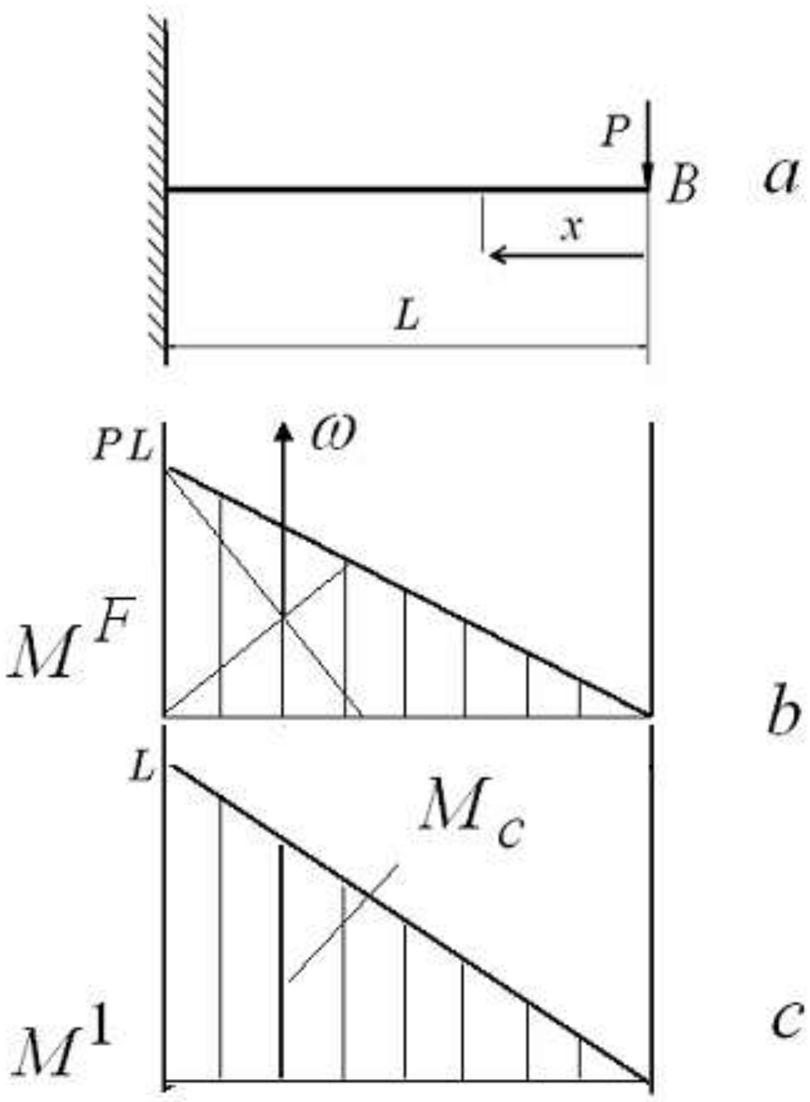

Fig. 17.1

On beginning

17.3. The second example of application

The given beam has the following geometric and forces parameters: $\ell = 0{,}5$ m, $\alpha = 0{,}5$, $\beta = 0{,}8$, $M = \alpha P\ell = 0{,}125$ Nm, $F = 0{,}5$ N (see Fig. 17.2).

Let us find the deflection of free end for given beam by Verescagin's rule.

At first, we will build the diagram of beanding moment M_F (Fig. 17.2):

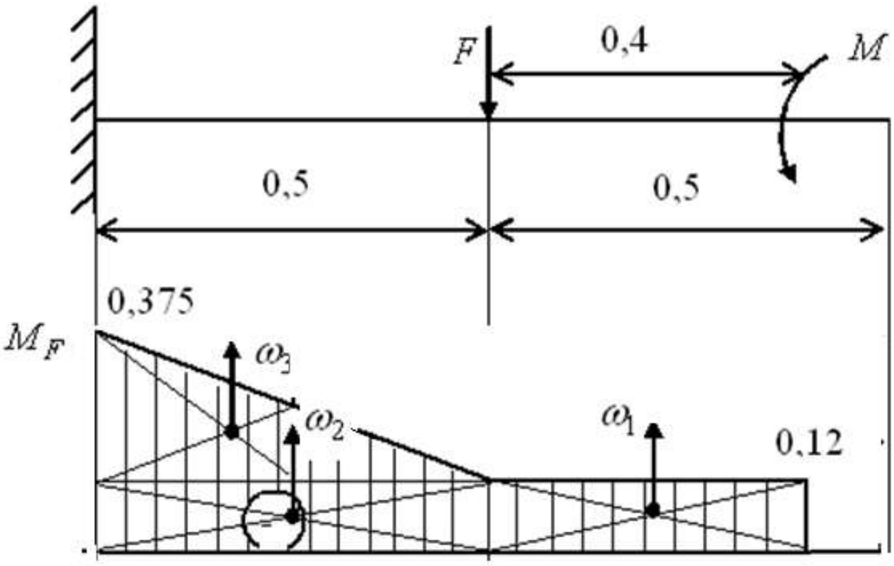

Fig. 17.2

I. portion $0 \le x_1 \le 0{,}1$ m. $\qquad M_{F_1} = 0$;

II portion $0{,}1 \text{ m} \le x_2 \le 0{,}5$ m. $\qquad M_{F_2} = -M = -0{,}125$ Nm;

III portion $0{,}5 \text{ m} \le x_3 \le 1$ m. $\qquad M_{F_3} = -M - F(x_3 - 0{,}5)$,

$$M(0{,}5) = -125 \text{ Nm}; \qquad M(1) = -0{,}375 \text{ Nm}.$$

Then, we remove the given external load and apply a single concentered force on the free end of the beam X_1 (Fig. 17.3).

Let us build the diagram of bending moment M^1:

I portion $0 \text{ m} \le x_1 \le 1$ m. $\qquad M^1 = -X_1 \cdot x_1$,

$$M(0) = 0, \qquad M(1) = 1 .$$

To use this formula (17.1), we divide the diagram M_F into simple figures ω_i, which will be two rectangles and a triangle (Fig. 17.2).

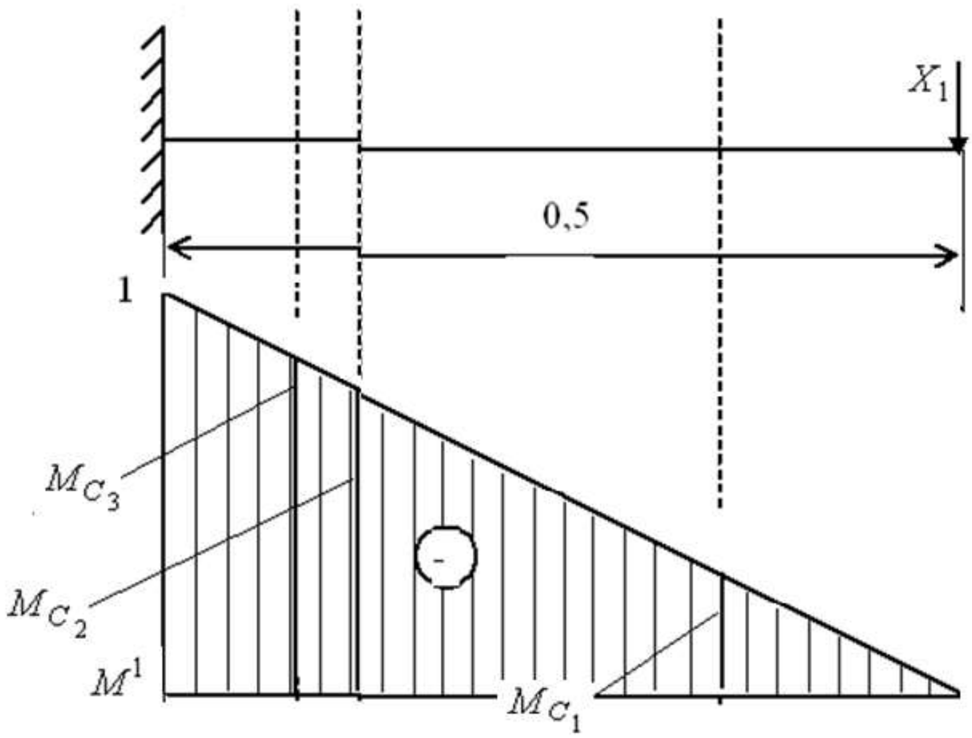

Fig. 17.3

To obtain values M_{C_i}, we will lower the lines from the centroid of each component ω_i on the diagram M^1 and the corresponding ordinate of this diagram will be parameter M_{C_i} (Fig. 17.3).

Thus, using Fig.17.2, we find the value of the areas ω_i:

$$\omega_1 = 0{,}4 \cdot (-0{,}125) = -0{,}05 \text{ Nm}^2;$$

$$\omega_2 = 0{,}5 \cdot (-0{,}125) = -0{,}0625 \text{ Nm}^2;$$

$$\omega_3 = 0{,}5 \cdot 0{,}5 \cdot (-0{,}25) = -0{,}0625 \text{ Nm}^2.$$

Then the corresponding ordinates of the centroids M_{C_i} will be equal:

$$M_{C_1} = -(0{,}1 + 0{,}5 \cdot 0{,}4) = -0{,}3 \text{ m};$$

$$M_{C_2} = -(1 - 0{,}25) = -0{,}75 \text{ m};$$

$$M_{C_3} = -(1 - 0,5 \cdot 0,33) \approx -0,835 \text{ m.}$$

Finally, substituting the values of all found quantities into Vereshchagin's formula, we get:

$$y = \frac{1}{EI}\{0,05 \cdot 0,3 + 0,0625 \cdot 0,75 + 0,0625 \cdot 0,835\} \approx$$

$$\approx \frac{0,114}{EI} \text{ (m).}$$

On beginning