Lecture 16
Double - integration method

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Plan of lection

- 1. Definition of beam deformation
- 2. The theory of double - integration method
- 3. The applying of double - integration method for a beam

Definition of beam deformation

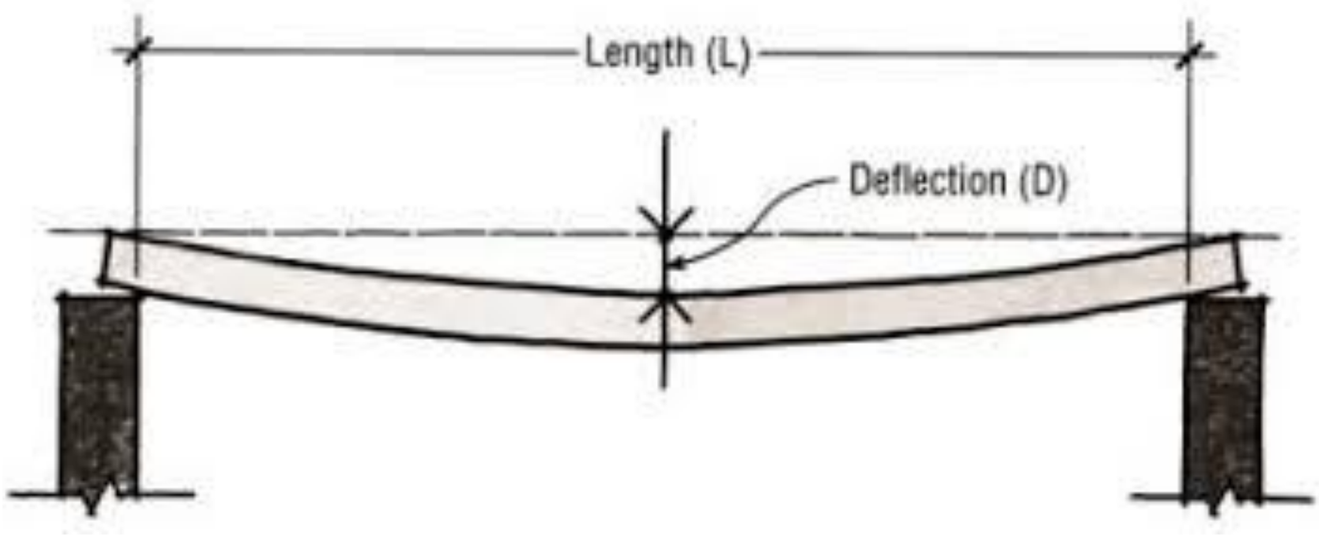

Fig. 1

Definition of beam deformation

The deflection is measured from the original neutral surface to the neutral surface of the deformed beam

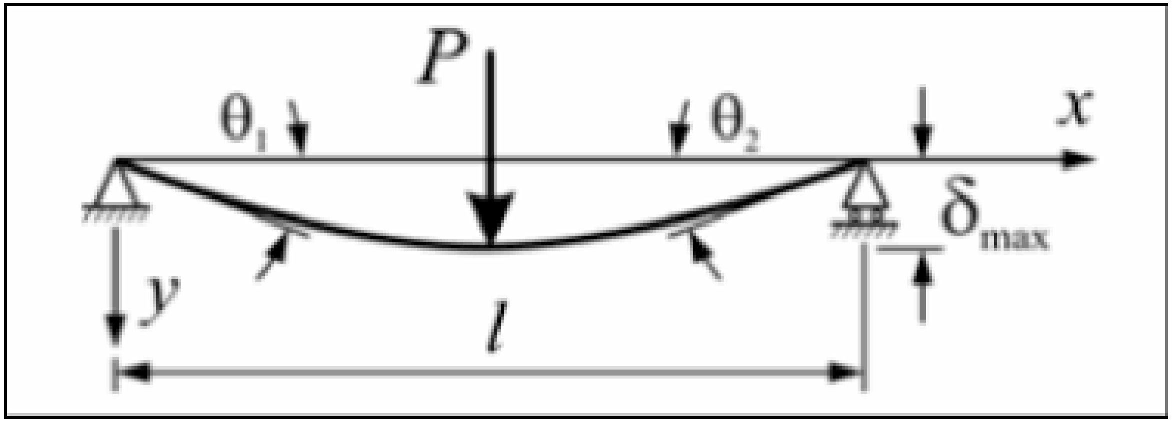

Fig. 2

Definition of beam deformation

In many building codes the maximum allowable deflection of a beam is not to exceed of the length of the beam.

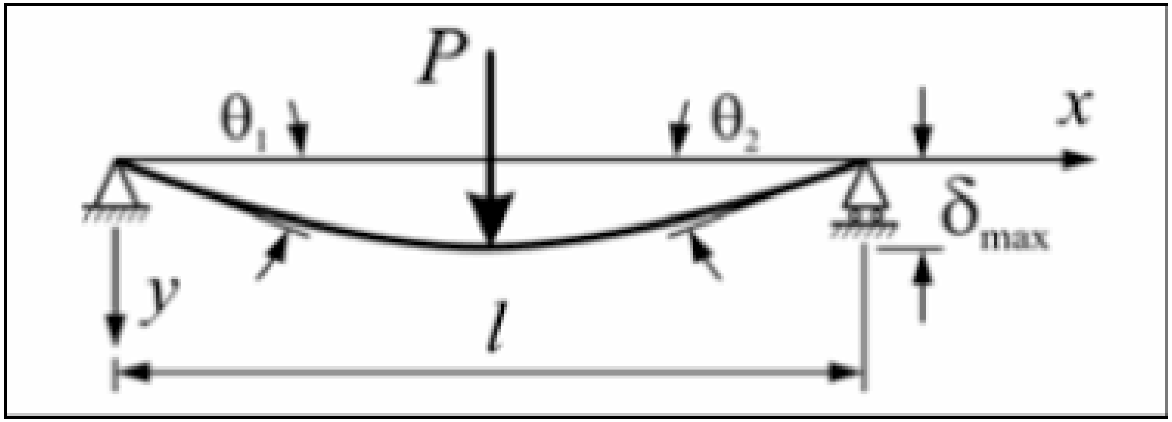

Fig. 2

The theory of double - integration method

The differential equation of the deflection curve of the bent beam is:

$$EI\frac{d^2y}{dx^2} = M_x \qquad (1)$$

where

y is the deflection of the beam;

E is the modulus of elasticity of the beam;

I is the moment of inertia of the beam cross section about the neutral axis;

M_x is the bending moment at the distance x from one end of the beam

The applying of double - integration method for a beam

Determine the deflection at every point or the cantilever beam subject to the single concentrated force P

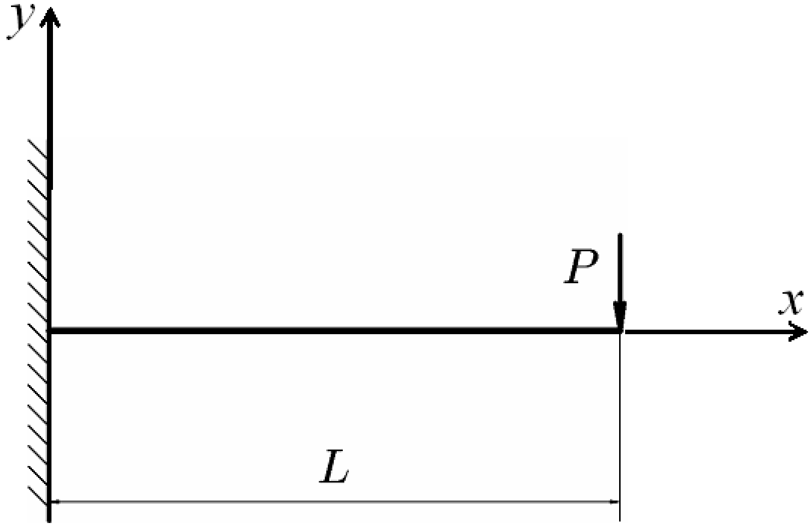

Fig. 3

The applying of double - integration method for a beam

The bending moment at any cross section a distance x from the wall is given by the sum of the moments of these two reactions about an axis through this section

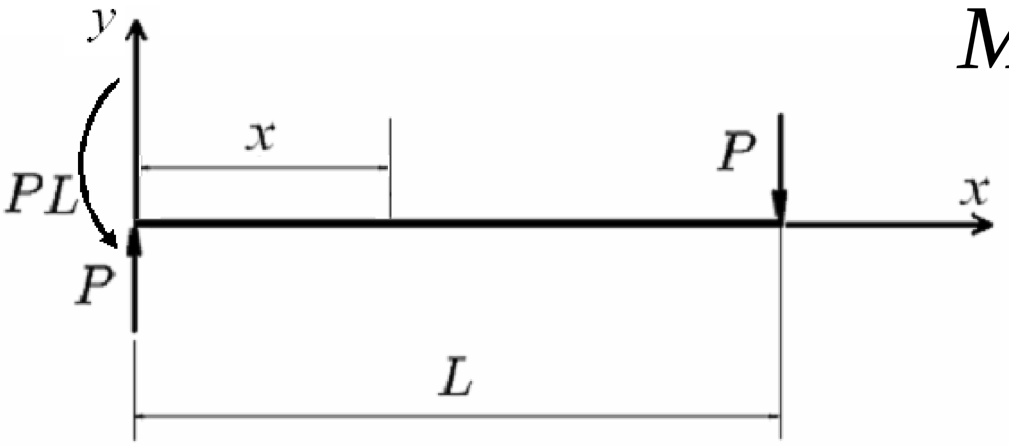

Fig. 4

$$M_x = -PL + P \cdot x \quad (2)$$

The applying of double - integration method for a beam

Substituting (2) in eq. (1), we obtain:

$$EI_x \frac{d^2y}{dx^2} = -PL + P \cdot x \qquad \textbf{(3)}$$

This equation is readily integrated once to yield:

$$EI_x \frac{dy}{dx} = -PLx + \frac{P \cdot x^2}{2} + C \qquad \textbf{(4)}$$

where C denotes a constant of integration

The applying of double - integration method for a beam

Using the first condition of support:

$$\frac{dy}{dx}\bigg|_{x=0} = 0 \qquad \textbf{(5)}$$

We get that:

$$0 = 0 + 0 + C$$

or

$$C = 0 \qquad \textbf{(6)}$$

The applying of double - integration method for a beam

Next integration of (3) yields:

$$EI_x y = -\frac{PLx^2}{2} + \frac{P \cdot x^3}{6} + D \qquad \textbf{(7)}$$

where D is a second constant of integration

Using the second condition of support:

$$y\big|_{x=0} = 0 \qquad \textbf{(8)}$$

There, at x=o, the deflection y is zero since the bar is rigidly clamped.

The applying of double - integration method for a beam

Substituting (7) in condition (8), we obtain:

$$0 = 0 + 0 + D$$

or:

$$D = 0 \qquad \textbf{(9)}$$

Then finally we get:

$$EI_x y = -\frac{PLx^2}{2} + \frac{P \cdot x^3}{6} \qquad \textbf{(10)}$$

The applying of double - integration method for a beam

The deflection is a maximum at the right end of the beam x=L, under the load P :

$$EI_x y_{\max} = -\frac{PL^3}{3} \qquad \textbf{(11)}$$

If only the magnitude of the maximum deflection at x=L is desired, we have:

$$\delta_{\max} = -\frac{PL^3}{3EI_x} \qquad \textbf{(12)}$$