Lecture 15
CALCULATION OF SIMPLY SUPPORTED BEAMS

Plan

1. The determination of the reactions of beam supports.
2. The building of diagrams of the shearing force and the bending moments for simply supported beam.
3. The selection of cross-section for given beam.

15.1.The determination of the reactions of beam supports.

Let us consider the simply supported beam shown in Fig. 15.1:

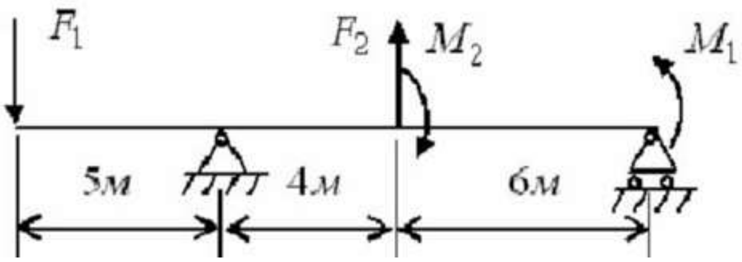

Fig. 15.1.

Taking into account that for given beam we have the following parameters: $[\sigma]=160$ MPa, $F_1=18$ kN, $F_2=30$ kN, $M_1=20$ kNm, $M_2=10$ kNm.

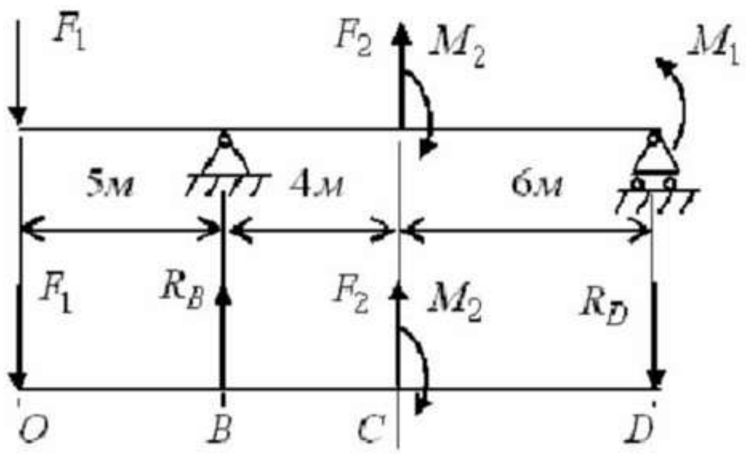

Fig. 15.2.

According to Fig. 15.2, from statics we have:

1. $\sum_{i=1}^{n} F_{ix} = 0;$ $\quad R_{Bx} \equiv 0.$

2. $\sum M_D = 0,$ $\quad -M_1 + F_2 \cdot CD + M_2 + R_{By} \cdot BD - F_1 \cdot OD = 0,$

or

$$R_{B_y} = \frac{M_1 - F_2 \cdot CD - M_2 + F_1 \cdot OD}{BD} = $$
$$= \frac{20 - 30 \cdot 6 - 10 + 18 \cdot 15}{10} = 10 \text{ kN};$$

3. $\sum M_B = 0,$ $\quad -M_1 - F_2 \cdot BC + M_2 - R_D \cdot BD - F_1 \cdot OB = 0.$

Then:

$$R_D = \frac{-M_1 - F_2 \cdot BC + M_2 - F_1 \cdot OB}{BD} =$$
$$= \frac{-20 - 30 \cdot 4 + 10 - 18 \cdot 5}{10} = -22 \text{ kN}.$$

Let us execute verification of obtained reactions:

4. $\sum_{i=1}^{n} F_{iy} = 0;$

$$-F_1 + R_{By} - R_D + F_2 = -18 + 10 - 22 + 30 = -8 + 8 = 0.$$

Thus, reactions are correctly determined.

On beginning

15.2. The building of diagrams of the shearing force and the bending moments for simply supported beam.

Let us divide a beam into portions by characteristic transversal sections O, B, C, D (Fig. 15.2).

Let us define the value of transversal force in characteristic sections and build the diagram Q_y (Fig. 15.3):

$$Q_{y_O}^{rt} = -F_1 = -18 \text{ kN}; \quad Q_{y_O}^{lt} = -F_1 = -18 \text{ kN};$$

$$Q_{y_B}^{rt} = -F_1 + R_B = -18 + 10 = -8 \text{ kN};$$

$$Q_{y_C}^{lt} = -F_1 + R_B = -18 + 10 = -8 \text{ kN};$$

$$Q_{y_C}^{rt} = -F_1 + R_B + F_2 = -18 + 10 + 30 = 22 \text{ kN};$$

$$Q_{y_D}^{lt} = -F_1 + R_B + F_2 = -18 + 10 + 30 = 22 \text{ kN}.$$

Let us define the value of bending moment in characteristic sections and build the diagram M_x (Fig. 15.3):

$$M_0 = 0; \qquad M_B = -F_1 \cdot AB = -18 \cdot 5 = -90 \text{ kNm};$$

$$M_B^{lt} = -F_1 \cdot OC + R_B \cdot BC = -18 \cdot 9 + 10 \cdot 4 = -122 \text{ kNm};$$

$$M_B^{lt} = -F_1 \cdot OC + R_B \cdot BC + M_2 = -18 \cdot 9 + 10 \cdot 4 + 10 = -112 \text{ kNm};$$

$$M_D^{lt} = -F_1 \cdot OD + R_B \cdot BD + M_2 + F_2 \cdot CD =$$
$$= -18 \cdot 15 + 10 \cdot 10 + 10 + 306 = 20 \text{ kNm}.$$

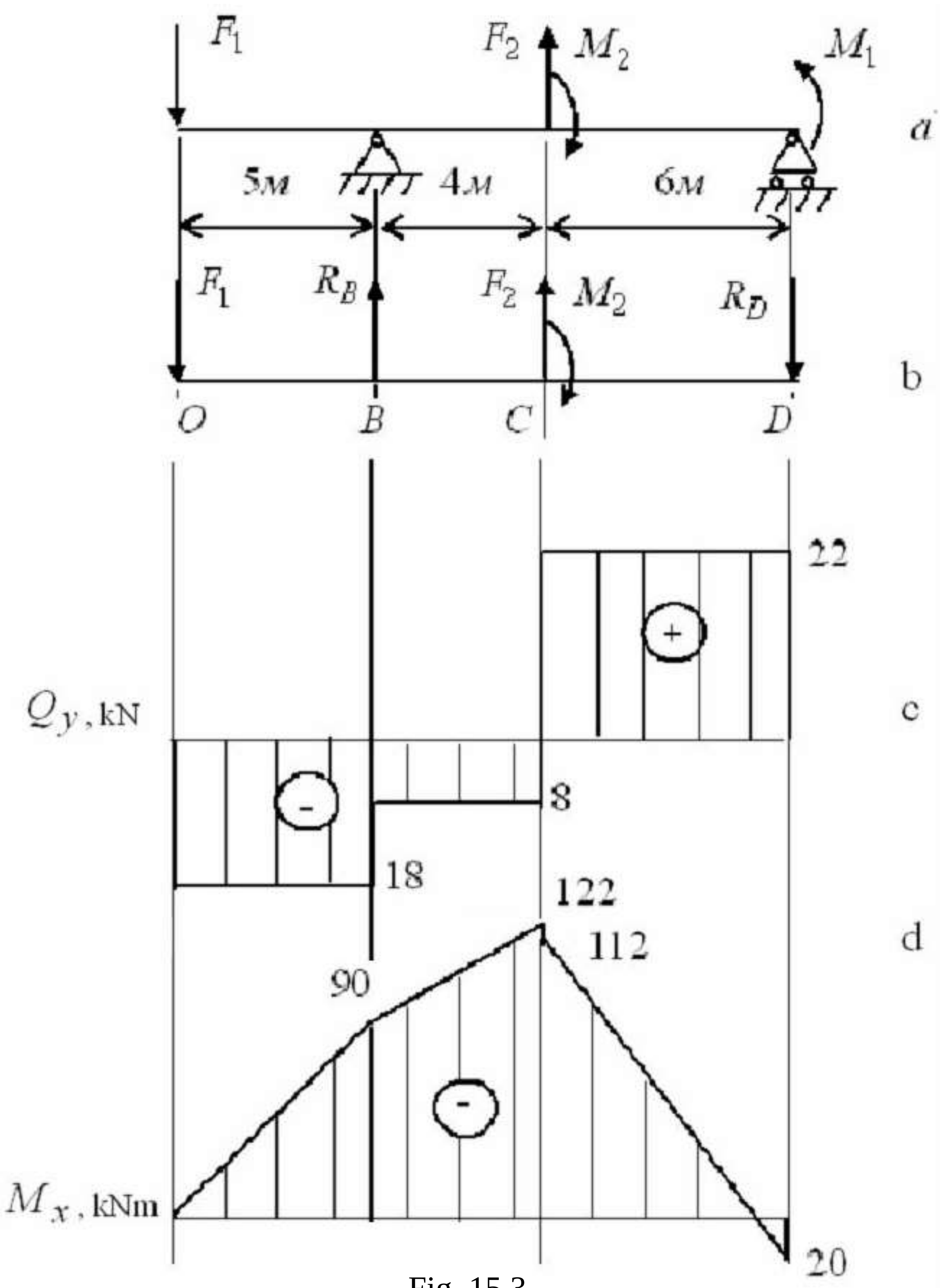

Fig. 15.3

On beginning

15.3. The selection of cross-section for given beam.

Let us calculate sizes transversal section of this beam from the conditions of durability at a bending for two cases:

a) the section of a rectangle cross - section:

$$W_x = \frac{M_{x\max}}{[\sigma]} = \frac{122 \cdot 10^3}{160 \cdot 10^6} = 0{,}762 \cdot 10^{-3}\ \text{m}^3.$$

Using a formula $W_x = \frac{bh^2}{6}$ and taking into account, that $h = 1{,}5b$, we find that:

$$b = \sqrt[3]{\frac{6W_x}{2{,}25}} = \sqrt[3]{\frac{6 \cdot 0{,}762 \cdot 10^6}{2{,}25}} = 127\ \text{mm};$$

b) the section of circle cross - section.

Using a formula $W_x = \frac{\pi d^3}{32}$, we find a diameter of cross - section:

$$d = \sqrt[3]{\frac{32W_x}{\pi}} = \sqrt[3]{\frac{32 \cdot 0{,}762 \cdot 10^6}{3{,}14}} = 196\ \text{mm}.$$

On beginning