Lecture 18
CASTIGLIANO'S THEOREM

Assos. Prof. A. Kutsenko

Plan of lection

- **1. General theory of Castigliano's theorem**
- **2. Example of application**

General theory of **Castigliano's theorem**

$$\delta_n = \int_\ell \frac{M}{GI_x}\frac{\partial M}{\partial P_n}dx \quad \textbf{(1)}$$

where

δ_n **is the displacement of beam point;**

P_n **is the fictitious force;**

General theory of **Castigliano's theorem**

The displacement of an elastic body under the point of application of any force, in the direction of that force, is given by the partial derivative of the total internal strain energy with respect to that force.

Example

Condition:

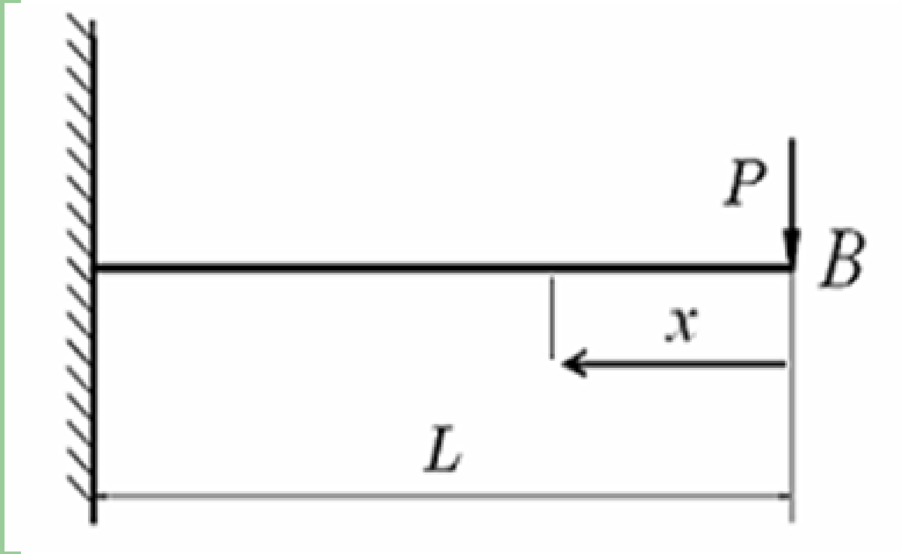

Fig. 1

Determine by Verescagin's rule the beam deflection at point **B**

Example

Hence the bending moment M at the section x is:

$$M_x = -P \cdot x \qquad \textbf{(2)}$$

Than the partial derivative of bending moment by force P is:

$$\frac{\partial M_x}{\partial P} = -x \qquad \textbf{(3)}$$

Example

Let us written the Castigliano's integral:

$$\delta_n = \int_{\ell} \frac{M}{GI_x} \frac{\partial M}{\partial P_n} dx$$

Substitute (2) and (3) into (1)

$$\delta_B = \int_0^L \frac{-P \cdot x}{EI_x}(-x)dx =$$

Example

or

$$\delta_B = \int_0^L \frac{P \cdot x^2}{EI_x} dx$$

Finally, we get:

$$\delta_B = \frac{P \cdot x^3}{3EI_x}\Bigg|_0^L = \frac{PL^3}{3EI_x}$$

Example

We obtained the result of problem, which is the analogical decision of problem in lectures 16 and 17.
It is differs only a sign, because the x -axis has opposite direction.