Lecture 28
COMBINED BENDING AND TORSION

Plan

1. General theory.
2. The building of diagrams of twisting and bending moments for shaft, which is working on combined bending and torsion.
3. Picking up of shafte diameter from strengt condition.

28.1 General theory

Most of the shafts are deformed on bending and twisting simultaneous. Usually they are the straight bars of circular or circular cross-section.

To calculate the shaft, we will be taking into account only the twisting and bending moments, which is acting in a dangerous cross-section. In this case we neglect the influence of shearing forces since the corresponding shear stresses are relatively small.

The maximum normal and tangential stresses for the round shafts are calculated by the formulas:

$$\sigma = \frac{M_з}{W_x}, \quad \tau = \frac{T}{W_\rho},$$

and for the round shafts we have equality $W\rho = 2W_x$.

In the case of simultaneous bending and twisting, the points of the cross-section of the shaft, which are most distant from the neutral axis, are dangerous.

According to the third theory of strength, we get:

$$\sigma_{eq} = \sqrt{\sigma^2 + 4\tau^2} = \sqrt{\left(\frac{M}{W_x}\right)^2 + 4\left(\frac{T}{W_\rho}\right)^2} = \sqrt{\left(\frac{M}{W_x}\right)^2 + 4\left(\frac{T}{2W_x}\right)^2} =$$
$$= \sqrt{\frac{M^2 + T^2}{W_x^2}}.$$

The expression in the numerator is called an equivalent moment:

$$M_{eq} = \sqrt{M^2 + T^2}. \quad (28.1)$$

Then the calculated formula for the circular shafts will look like:

$$\sigma_{eq} = \frac{M_{eq}}{W_x} \le [\sigma], \tag{28.2}$$

usually the shafts are made from material that has the property:

$$\lfloor \sigma_p \rfloor = [\sigma_c] = [\sigma].$$

According to the formula (28.2), the round shafts analogies a beam are calculated on bending. But in this case we don't use the bending moment. We must use an equivalent moment.

Applying the energy theory of strength, we obtain:

$$\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2} = \sqrt{\left(\frac{M}{W_x}\right)^2 + 3\left(\frac{T}{W_\rho}\right)^2} = \sqrt{\left(\frac{M}{W_x}\right)^2 + 3\left(\frac{T}{2W_x}\right)^2} =$$

$$= \sqrt{\frac{M^2 + 0{,}75T^2}{W_x^2}}.$$

namely

$$M_{eq} = \sqrt{M^2 + 0{,}75T^2}. \tag{28.3}$$

The diagrams of T; M_{hor}; M_{vert} often built in axonometric. The bolts and fastening screws experience the simultaneous deformations of torsion and tension. The drills and spindles of drilling machines deformed simultaneous on torsion and compression. These parts are usually made of materials, which have the property:

$$\lfloor \sigma_p \rfloor = [\sigma_c] = [\sigma].$$

Normal and maximum tangential stresses in these cases are calculated by the formulas:

$$\sigma = \frac{F}{A}, \qquad \tau = \frac{T}{W_\rho}.$$

Applying the third theory of strength, we find the calculation formula:

$$\sigma_{eq} = \sqrt{\left(\frac{F}{A}\right)^2 + 4\left(\frac{T}{W_\rho}\right)^2} \le [\sigma]. \qquad (28.4)$$

Applying the energy theory of strength, we obtain:

$$\sigma_{eq} = \sqrt{\left(\frac{F}{A}\right)^2 + 3\left(\frac{T}{W_\rho}\right)^2} \le [\sigma]. \qquad (28.5)$$

On beginning

28.2. The building of diagrams of twisting and bending moments for shaft, which is working on combined bending and torsion.

Let the shaft, that is shown in fig. 28.1, has a pulley with a diameter $D_1 = 0{,}9$ m, which is with the angle of the belt to the horizon $\alpha_1 = 45^0$. This pulley works $n = 400$ revolutions per minute, and transmits power $N = 50$ kilowatt. The other two pulleys have the same diameter $D_2 = 0{,}3$ m and the same angles of inclination of the straps to the horizon $\alpha_2 = 45^0$. Each of them is transmitting the power $\frac{N}{2}$. The distance between the pulleys, respectively, are: $a = 1{,}2$ m, $b = c = 0{,}5$ m. It is also necessary to take into account that $T_1 = 2t_1$, $T_2 = 2t_2$, according to fig. 28.2.

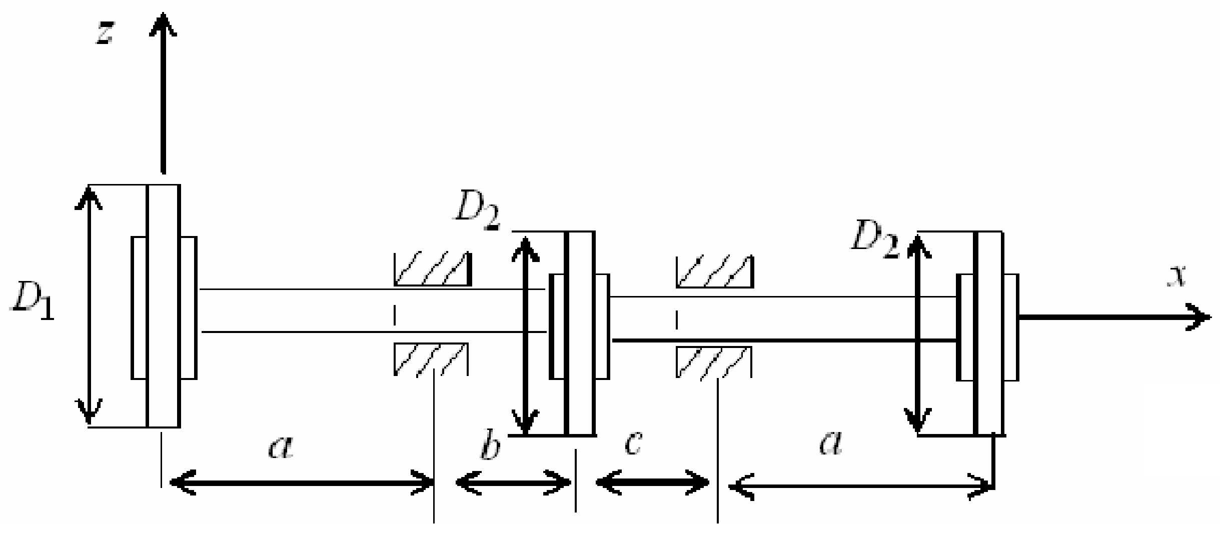

Fig. 28.1

Pick the diameter of the shaft, taking into account that $[\sigma] = 70$ kPa and round it to the standard.

Let us determine the moments attached to the pulleys. To do this, we find the value of the angular velocity of the shaft by the formula:

$$\omega = \frac{2\pi \cdot n}{60} = \frac{2 \cdot 3{,}14 \cdot 400}{60} \approx 41{,}9 \text{ rad/s}.$$

Then the torque applied to the first pulley is equal:

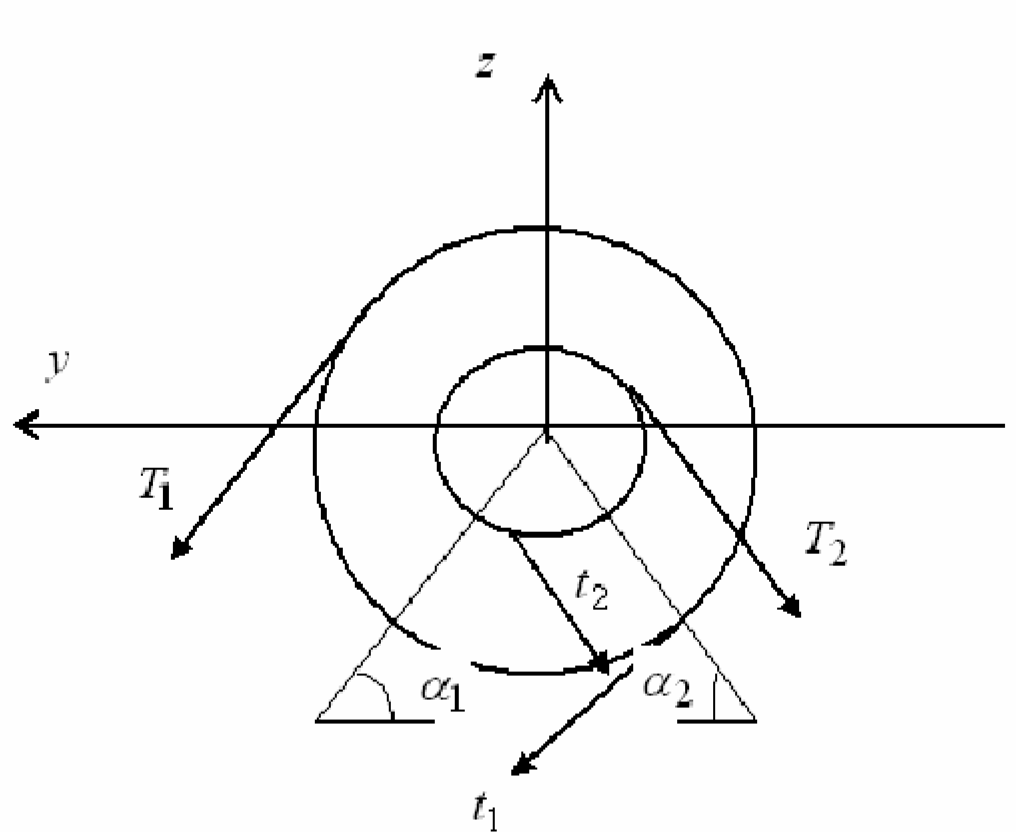

Fig. 28.2

$$M_1 = \frac{N}{\omega} = \frac{50000}{41{,}9} = 1193 \text{ Nm},$$

accordingly, the torque M_2 will be equal:

$$M_2 = \frac{N}{2\omega} = \frac{50000}{2 \cdot 41{,}9} = 596{,}5 \text{ Nm}.$$

The corresponding diagram of torque is depicted in Fig. 28.3.

Determine the effort t_1 and t_2, which are acting on the pulley of the shaft, we obtain:

$$t_1 = \frac{M_1}{D_1} = \frac{1193}{0{,}9} \approx 1326 \text{ N},$$

$$t_2 = \frac{M_2}{D_2} = \frac{596{,}5}{0{,}3} \approx 1990 \text{ N}.$$

Taking into account the condition of the problem, the resultants of the system of the forces T_i and t_i, will be equal:

$$R_1 = T_1 + t_1 = 3t_1 = 3978 \text{ N}, \quad R_2 = T_2 + t_2 = 3t_2 = 5970 \text{ N}.$$

Diagram of M_k Nm

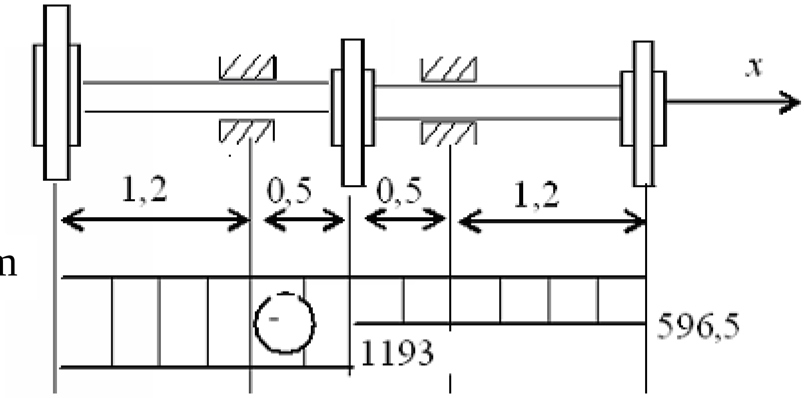

Fig. 28.3

Determine the horizontal and vertical components of forces P_1 and P_2 according to the scheme of their location (Fig. 28.2):

$$H_1 = R_1 \cdot \cos\alpha_1 = 2813 \text{ N}, \quad H_2 = -R_2 \cdot \cos\alpha_2 = -4221 \text{ N}.$$

$$V_1 = -R_1 \cdot \sin\alpha_1 = -2813 \text{ N}, V_2 = -R_2 \cdot \sin\alpha_2 = -4221 \text{ N}.$$

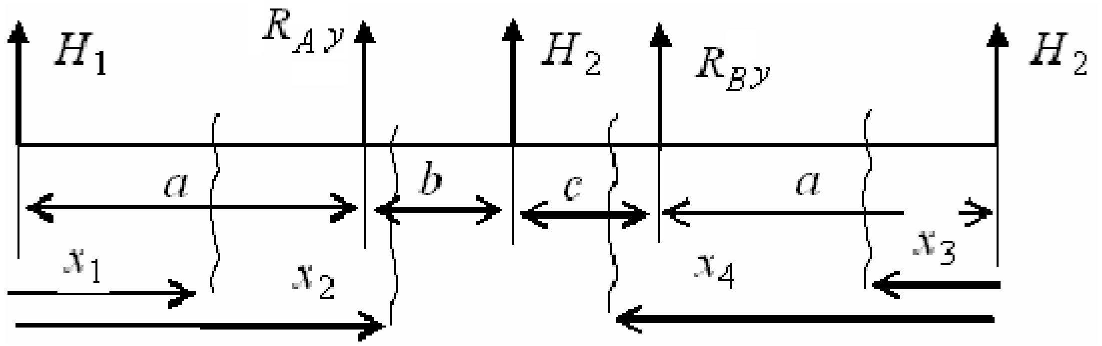

Fig. 28.4

Let us load the shaft with these forces as a beam that is lying on two supports. We find the reaction of the supports and build the diagrams of bending moments in the corresponding planes.

Load the shaft with horizontal forces (Fig. 28.4) and find the reaction of its supports, we get:

$$\sum M_{B_i} = 0, \qquad R_{Ay} = \frac{-H_1(a+b+c)+H_2(a-c)}{b+c} = -9143{,}3 \text{ N};$$

$$\sum M_{A_i} = 0, \qquad R_{By} = \frac{H_1 a - H_2(a+2b+c)}{b+c} = 14772{,}3 \text{ N}.$$

Verification of the reactions of supports of beams, that was determinated:

$$\sum F_{yi} = 0: \quad H_1 + 2H_2 + R_{Ay} + R_{By} =$$
$$= 2813 - 2 \cdot 4221 - 9143{,}3 + 14772{,}3 = -5629 + 5629 = 0.$$

We construct the diagram of the bending moment M_y (Fig. 28.5):

I portion. $0 \le x_1 \le 1{,}2$ m. $\qquad M_{y_1} = H_1 \cdot x_1,$

$$M(0) = 0, \qquad M(1{,}2) = 3376 \text{ Nm};$$

II portion. $1{,}2 \text{ m} \le x_2 \le 1{,}7 \text{ m}.$ $\quad M_{y_2} = H_1 \cdot x_2 + R_A^y \cdot (x_2 - 1{,}2),$

$$M(1{,}7) = 211 \text{ Nm};$$

III portion. $0 \le x_3 \le 1{,}2 \text{ m}.$ $\quad M_{y_3} = H_2 \cdot x_3,$

$$M(0) = 0, \qquad M(1{,}2) = -2111 \text{ Nm};$$

IV portion. $1{,}2 \text{ m} \le x_4 \le 1{,}7 \text{ m}.$ $\quad M_{y_4} = H_2 \cdot x_4 + R_B^y \cdot (x_4 - 1{,}2),$

$$M(1{,}7) = 211 \text{ Nm}.$$

Diagram of M_y, Nm

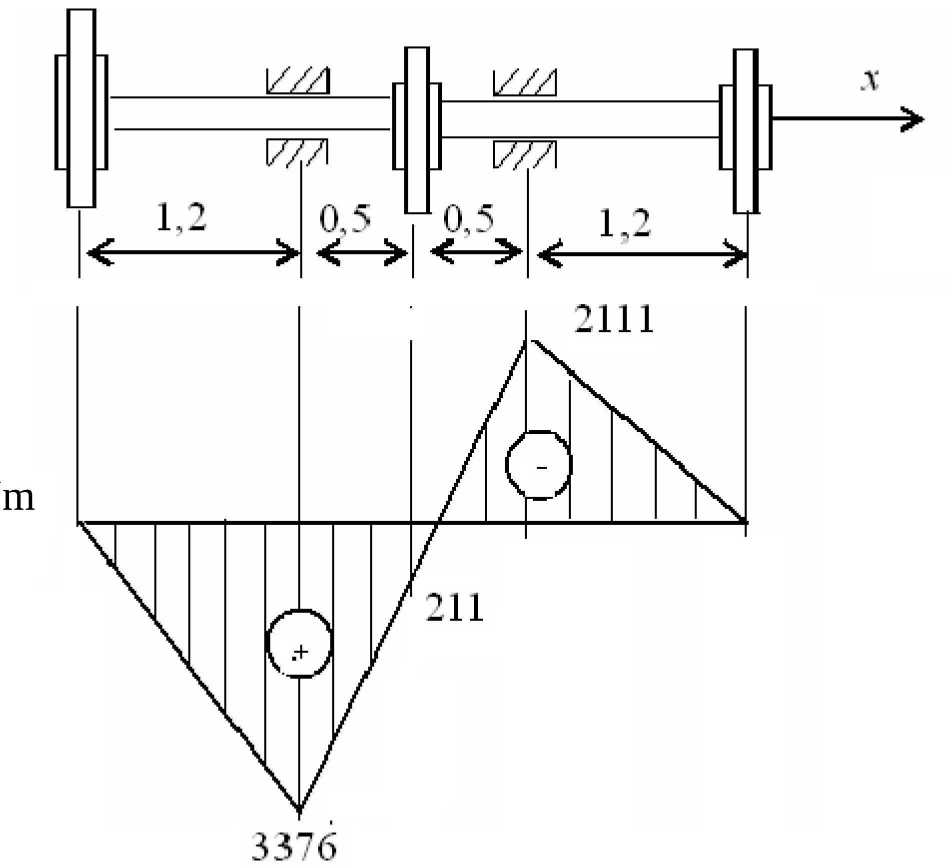

Fig. 28.5

Load the shaft with vertical forces (Fig. 28.6) and find the reaction of its supports, we get:

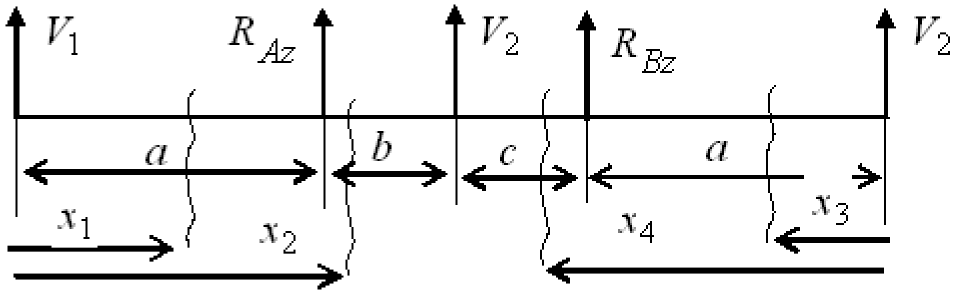

Fig. 28.6

$$\sum M_{B_i} = 0, \qquad R_{Az} = \frac{-V_1(a+b+c)+V_2(a-c)}{b+c} = 3233{,}9 \text{ N};$$

$$\sum M_{A_i} = 0, \qquad R_{Bz} = \frac{V_1 a - V_2(a+2b+c)}{b+c} = 8021{,}1 \text{ N}.$$

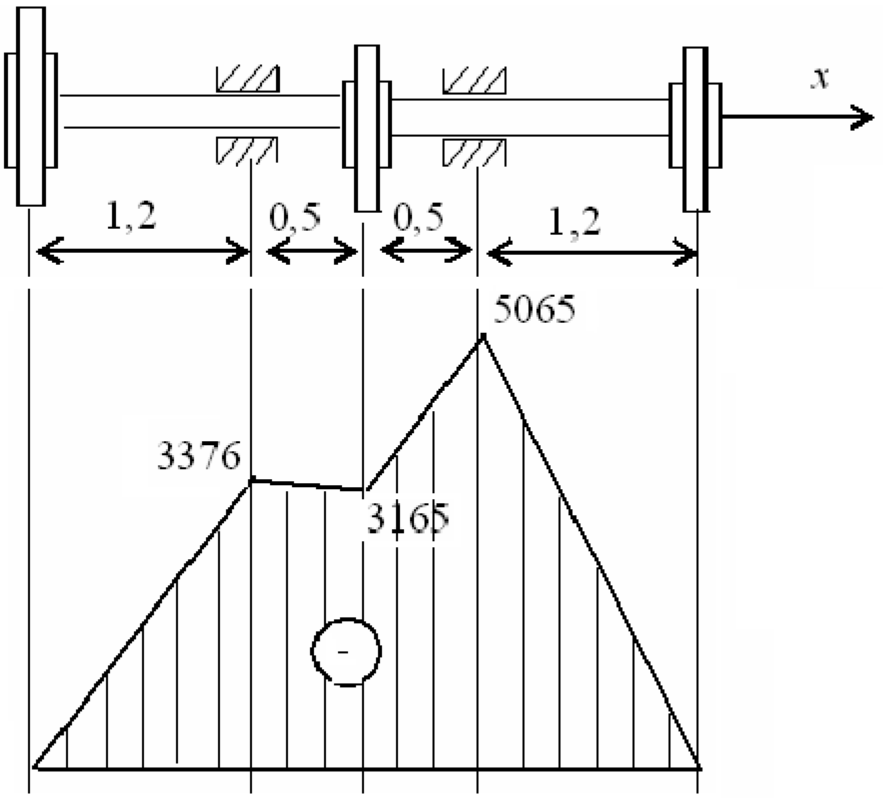

Diagram of M_z, Nm

Fig. 28.7

Verification of the reactions of supports of beams, that was determinated:

$$\sum F_{zi} = 0: \quad V_1 + 2V_2 + R_{Az} + R_{Bz} =$$
$$= -2813 - 2 \cdot 4221 + 3233{,}9 + 8021{,}1 = -11255 + 11255 = 0.$$

We construct the diagram of the bending moment M_z (Fig. 28.7):

I portion. $0 \le x_1 \le 1{,}2$ m. $\qquad M_{z_1} = V_1 \cdot x_1,$

$$M(0) = 0, \qquad M(1{,}2) = -3376 \text{ Nm};$$

II portion. $1{,}2 \text{ m} \le x_2 \le 1{,}7$ m. $\qquad M_{z_2} = V_1 \cdot x_2 + R_A^z \cdot (x_2 - 1{,}2),$

$$M(1{,}7) = -3165 \text{ Nm};$$

III portion. $0 \le x_3 \le 1{,}2$ m. $\qquad M_{z_3} = V_2 \cdot x_3,$

$$M(0) = 0, \qquad M(1{,}2) = -5065 \text{ Nm};$$

IV portion. $1{,}2 \text{ m} \le x_4 \le 1{,}7$ m. $\qquad M_{z_4} = V_2 \cdot x_4 + R_B^z \cdot (x_4 - 1{,}2),$

$$M(1{,}7) = -3165 \text{ Nm}.$$

Finding bending moments in cross-sections and constructing a diagram of the total bending moment (Fig. 28.8):

$$M_{t_1} = \sqrt{M_{y_1}^2 + M_{z_1}^2} = \sqrt{3376^2 + (-3376)^2} = 4774 \text{ Nm},$$

$$M_{t_2} = \sqrt{M_{y_2}^2 + M_{z_2}^2} = \sqrt{211^2 + (-3165)^2} = 3172 \text{ Nm},$$

$$M_{t_3} = \sqrt{M_{y_3}^2 + M_{z_3}^2} = \sqrt{(-2111)^2 + (-5065)^2} = 5487 \text{ Nm}.$$

Diagram of M_t, Nm

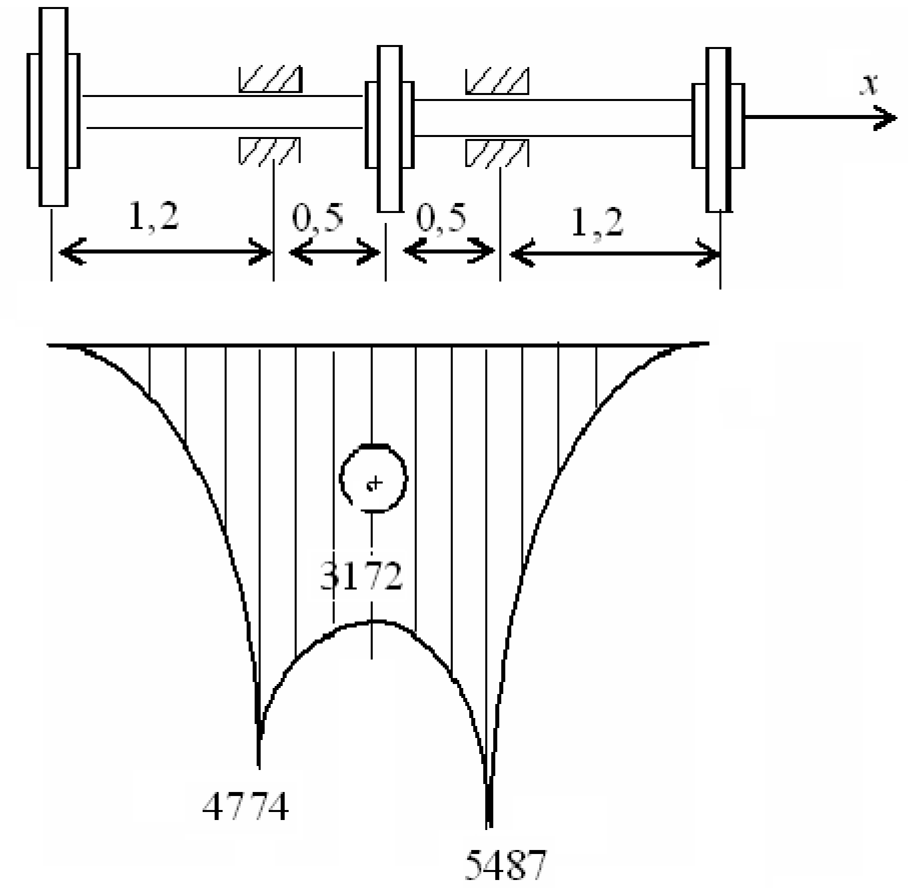

Fig. 28.8

On beginning

28.3. Picking up of shafte diameter from strengt condition.

Determine the calculated moment for the third theory of strength:

$$M_{III} = \sqrt{M_{t_i}^2 + M_{\kappa_i}^2} \, ,$$

or:

$$M_{III} = \sqrt{5487^2 + 596{,}5^2} = 5519{,}3 \text{ Nm.}$$

The diameter of the shaft is determine from the condition of strength, we obtain:

$$d \geq \sqrt[3]{\frac{32 \cdot M_{III}}{\pi \cdot [\sigma]}} = \sqrt[3]{\frac{32 \cdot 5519{,}3}{3{,}14 \cdot 70 \cdot 10^3}} \approx 0{,}93 \text{ m} = 93 \text{ mm.}$$

We accept a diameter equal to 100 mm.

On beginning