Lecture 27
THE OF-CENTRE ACTING OF FORCE OF TENSION OR COMPRESSION

Assos. Prof. A. Kutsenko

Plan of lection

- 1. General theory
- 2. The core of the cross-section
- 3. Example of calculation of beam on the off-centre acting of force

General theory

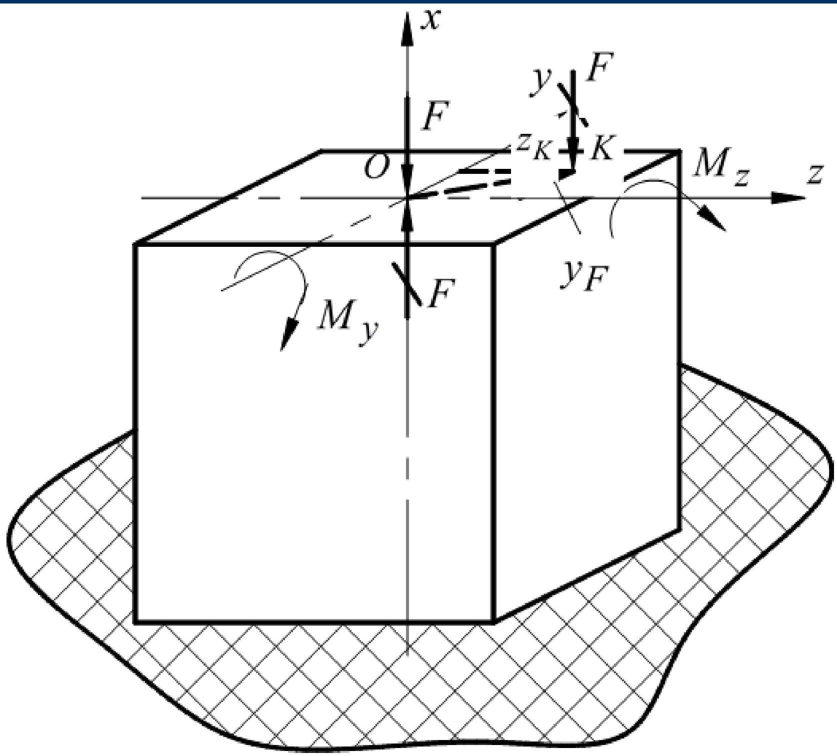

Fig. 1

$$M_O = F \cdot OK$$

To decompose the moment into two components

$$M_z = F \cdot y_F$$

$$M_y = F \cdot z_F \qquad (1)$$

General theory

The stresses are determined from the dependence:

$$\sigma = -\frac{F}{A} - \frac{M_z \cdot y}{I_z} - \frac{M_y \cdot z}{I_y} \qquad \textbf{(2)}$$

or

$$\sigma = -\frac{F}{A} - \frac{F \cdot y_F \cdot y}{I_z} - \frac{F \cdot z_F \cdot z}{I_y} \qquad \textbf{(3)}$$

In the case of acting a tensile force :

$$\sigma = \frac{F}{A} + \frac{F \cdot y_F \cdot y}{I_z} - \frac{F \cdot z_F \cdot z}{I_y} \qquad \textbf{(4)}$$

General theory

In general case we get:

$$\sigma = \pm \frac{F}{A} \pm \frac{F \cdot y_F \cdot y}{I_z} \pm \frac{F \cdot z_F \cdot z}{I_y} \quad \textbf{(5)}$$

Given that

$$i_z = \sqrt{\frac{I_z}{A}} \qquad i_y = \sqrt{\frac{I_y}{A}}$$

The formula (5) can be represented as:

$$\sigma = \pm \frac{F}{A} \pm \frac{F \cdot y_F \cdot y}{i_z^2 \cdot A} \pm \frac{F \cdot z_F \cdot z}{i_y^2 \cdot A}$$

General theory

or

$$\sigma = \frac{F}{A}\left(\pm 1 \pm \frac{y_F \cdot y}{i_z^2} \pm \frac{z_F \cdot z}{i_y^2}\right) \quad \textbf{(6)}$$

Conditions strength of materials:

$$\sigma_{\max} = \frac{F}{A}\left(\pm 1 \pm \frac{y_F \cdot y}{i_z^2} \pm \frac{z_F \cdot z}{i_y^2}\right) \le [\sigma_p]$$

(7)

$$\sigma_{\max} = -\frac{F}{A}\left(\pm 1 \pm \frac{y_F \cdot y}{i_z^2} \pm \frac{z_F \cdot z}{i_y^2}\right) \le [\sigma_{cm}]$$

The core of the cross-section

To define the location of the neutral axis of the cross section

$$1+\frac{y_F \cdot y_0}{i_z^2}+\frac{z_F \cdot z_0}{i_y^2}=0 \qquad \textbf{(8)}$$

let us suppose, that $y_0 = 0$, then $z_0 = a_z$:

$$a_z = -\frac{i_y^2}{z_F} \qquad \textbf{(9)}$$

The core of the cross-section

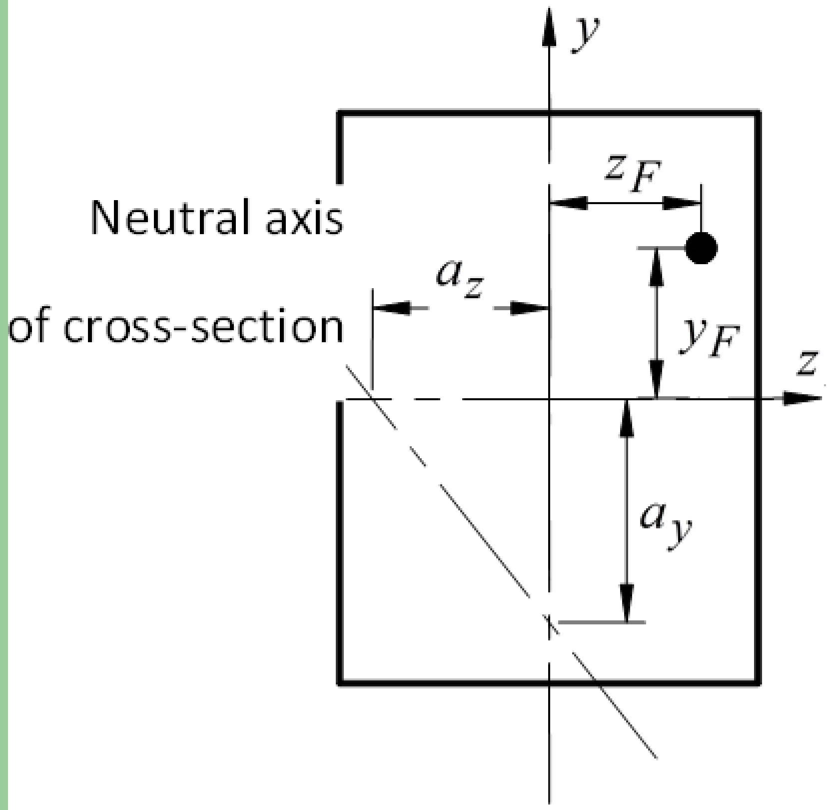

Fig. 2

For the case when $z_0 = 0$

$$a_y = -\frac{i_z^2}{y_F} \qquad \textbf{(10)}$$

The central part of the cross-section, in which or at its limit, the application of compressive force, causes only compressive stresses at all points of the cross-section is called the core of the cross-section.

The core of the cross-section

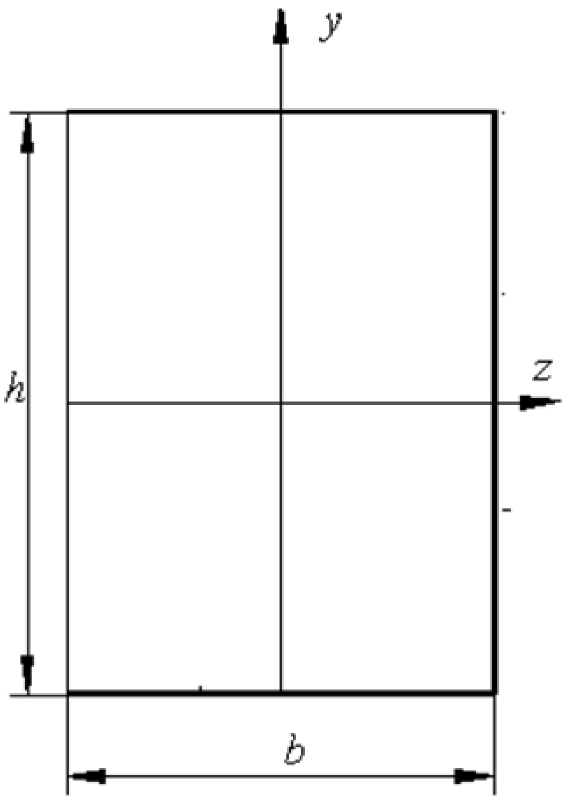

Fig. 3

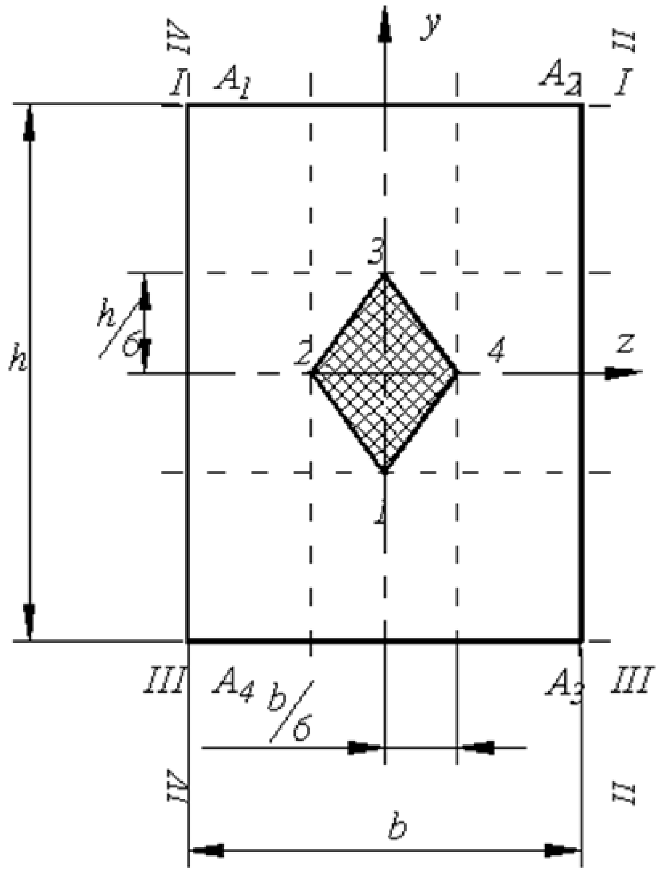

Fig. 4

Example of calculation

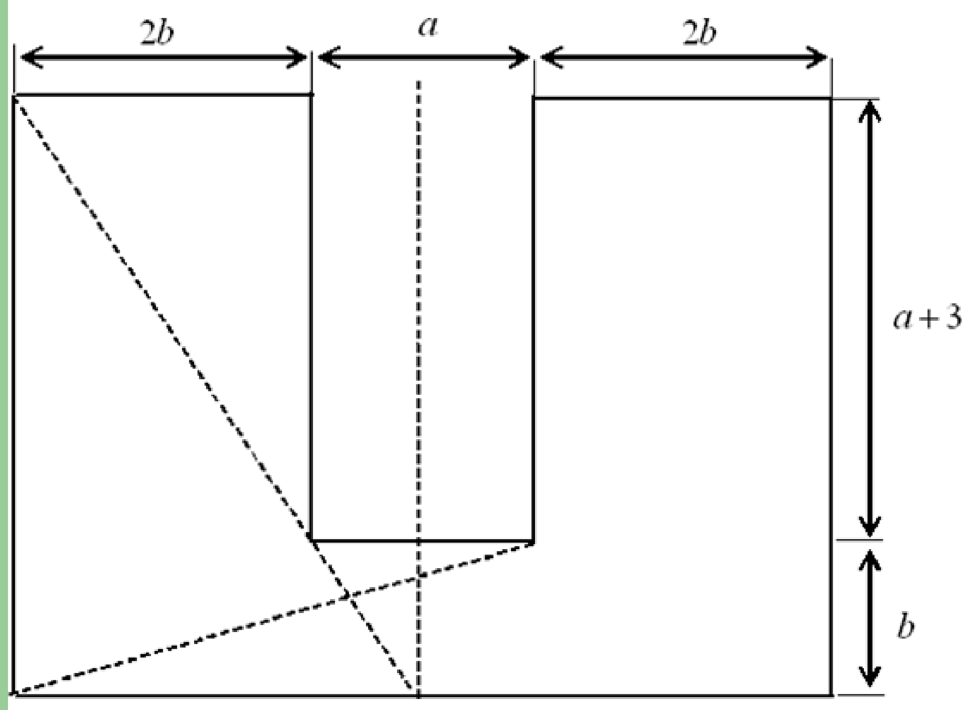

Fig. 6

Initial conditions:

$$a = 3 \text{ dm}$$

$$b = 2 \text{ dm}$$

To build the core of the cross-section

Example of calculation

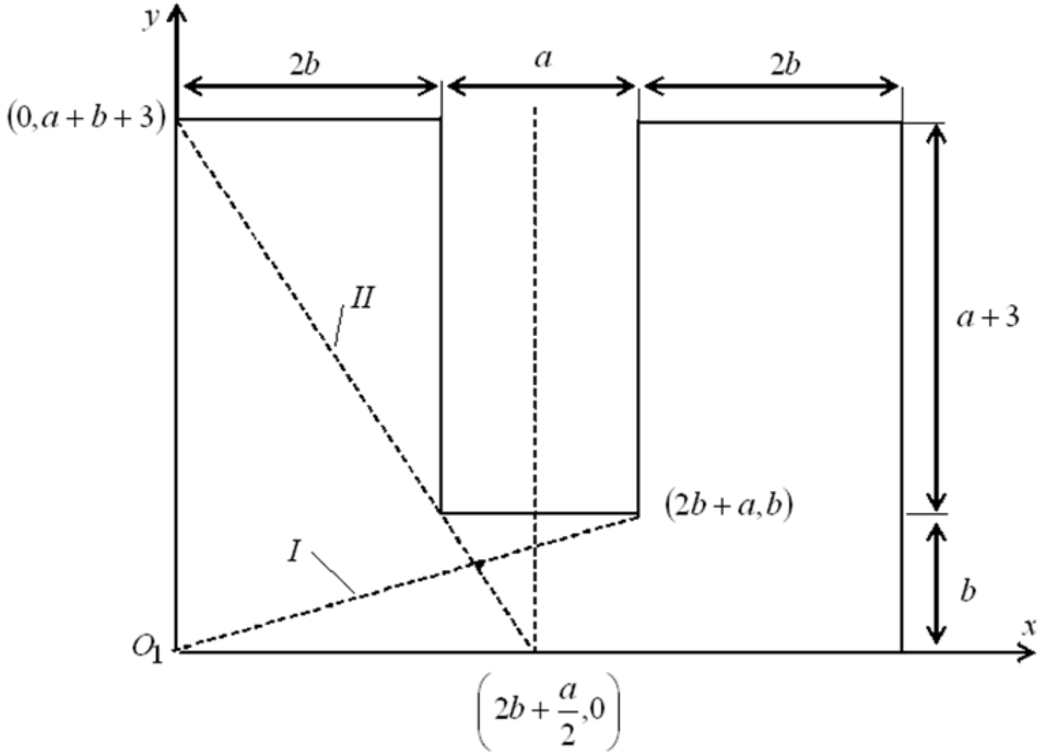

Fig. 7

To find the coordinates of the force application point:

$$\begin{cases} y = \dfrac{b}{2b+a}x \\ y = -\dfrac{2a+2b+6}{4b+a}x + (a+b+3) \end{cases}$$

Example of calculation

The solution of system

$$x = \frac{(2b+a)(4b+a)(b+a+3)}{8b^2+7ab+2a^2+12b+6a}$$

$$y = \frac{b(4b+a)(b+a+3)}{8b^2+7ab+2a^2+12b+6a}$$

finally, we obtain the coordinates of the point of application of force F:

$x_F = 4{,}597$ **dm** $y_F = 1{,}313$ **dm**

Example of calculation

Let us determine the position of the centroid of a given cross-section

$A_1 = 32$ dm²

$A_2 = 6$ dm²

$A_3 = 32$ dm²

$I_u = 392{,}705$ dm⁴

$I_v = 873{,}8$ dm⁴

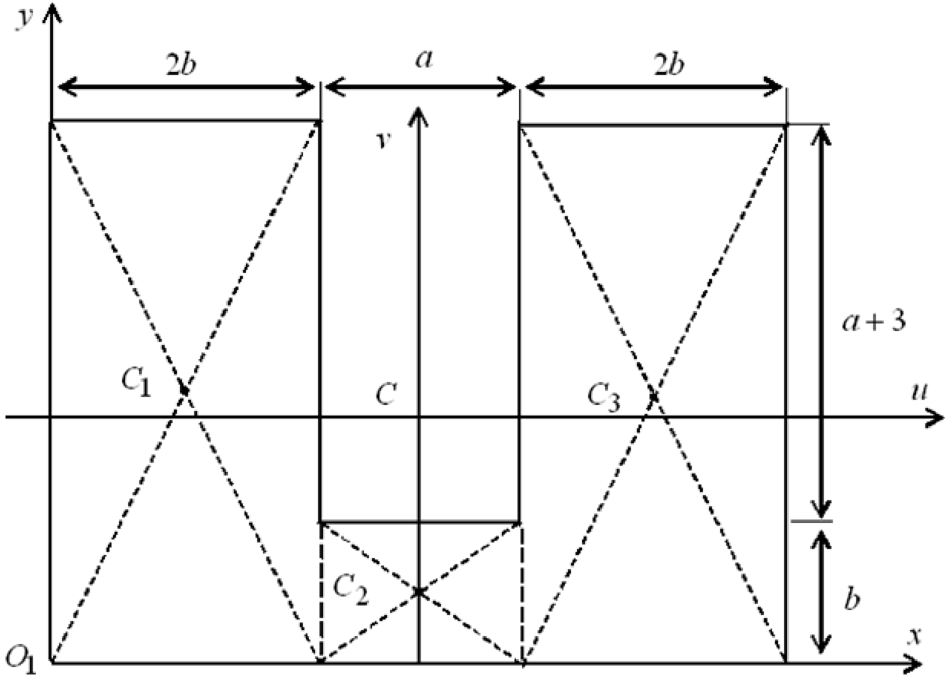

Fig. 9

Example of calculation

To calculate the squares of the radii of inertia relative to the axes u and v:

$$i_u^2 = \frac{I_u}{A} = \frac{392{,}705}{70} = 5{,}61 \text{ dm}^2 \qquad i_v^2 = \frac{I_v}{A} = \frac{873{,}8}{70} = 12{,}48 \text{ dm}^2$$

Then we obtain:

$$u_F = -\frac{i_v^2}{a_u} = -\frac{12{,}48}{a_u}$$

$$v_F = -\frac{i_u^2}{a_v} = -\frac{5{,}61}{a_v}$$

Example of calculation

Let us calculate the coordinates of the core of the cross-section:

1. $a_u = 5{,}5$ dm $\qquad u_F = -\frac{12{,}48}{5{,}5} = -2{,}27$ dm

2. $a_v = 4{,}257$ dm $\qquad v_F = -\frac{5{,}61}{4{,}257} = -1{,}32$ dm

3. $a_u = -5{,}5$ dm $\qquad u_F = -\frac{12{,}48}{-5{,}5} = 2{,}27$ dm

4. $a_v = -3{,}74$ dm $\qquad v_F = -\frac{5{,}61}{-3{,}74} = 1{,}5$ dm

Example of calculation

The core of the cross-section

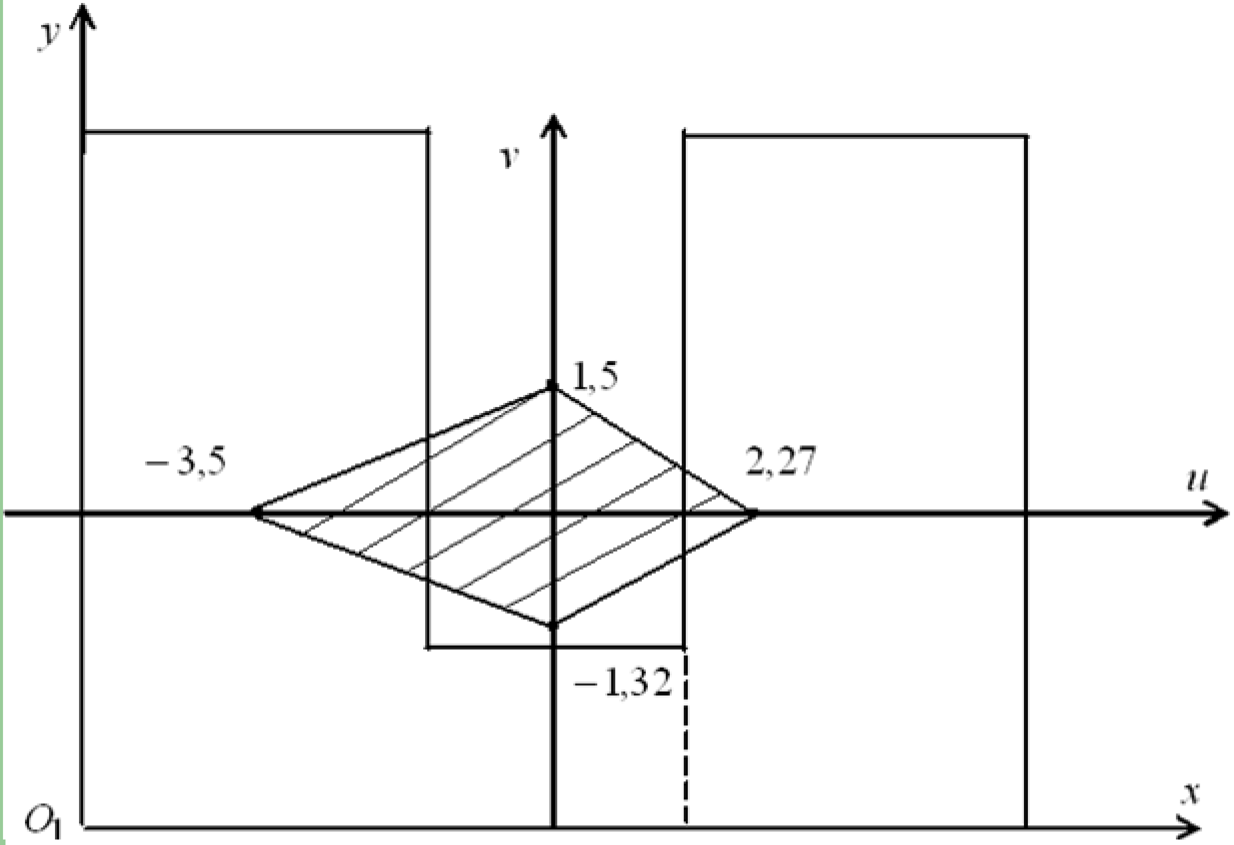

Fig. 10