Lecture 28
COMBINED BENDING AND TORSION

Assos. Prof. A. Kutsenko

Plan of lecture

- **1.** General theory
- 2. The building of diagrams of twisting and bending moments for shaft, which is working on combined bending and torsion
- 3. Picking up of shafte diameter from strengt condition

General theory

The maximum normal and tangential stresses for the round shafts are calculated by the formulas:

$$\sigma = \frac{M_з}{W_x} \qquad \tau = \frac{T}{W_\rho}$$

According to the third theory of strength, we get

$$\sigma_{eq} = \sqrt{\sigma^2 + 4\tau^2} = \sqrt{\frac{M^2 + T^2}{W_x^2}}$$

The expression in the numerator is called an equivalent moment:

$$M_{eq} = \sqrt{M^2 + T^2} \qquad \textbf{(1)}$$

General theory

Then the calculated formula for the circular shafts will look like:

$$\sigma_{eq} = \frac{M_{eq}}{W_x} \leq [\sigma] \qquad \textbf{(2)}$$

Applying the energy theory of strength, we obtain

$$\sigma_{eq} = \sqrt{\sigma^2 + 4\tau^2} = \sqrt{\frac{M^2 + 0{,}75T^2}{W_x^2}}$$

In that case the expression for an equivalent moment is:

$$M_{eq} = \sqrt{M^2 + 0{,}75T^2} \qquad \textbf{(3)}$$

General theory

Applying the third theory of strength, we find the calculation formula:

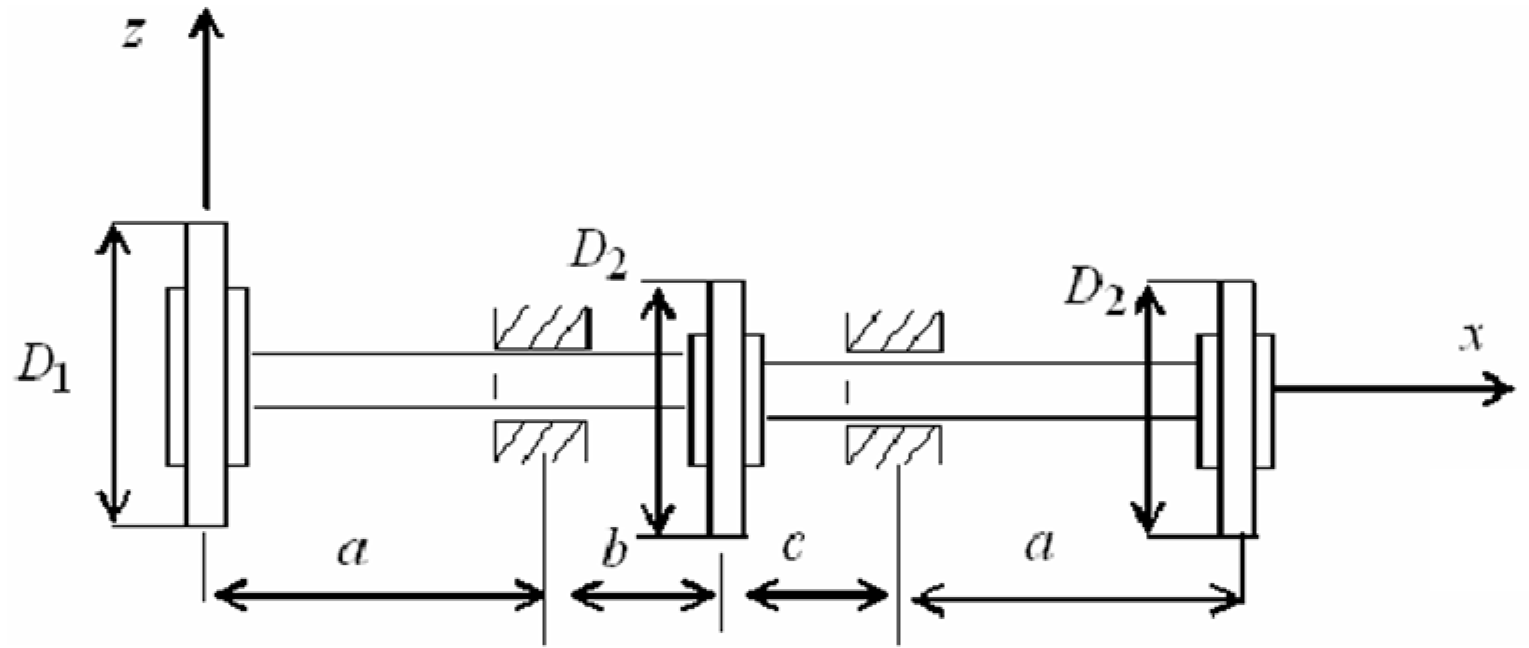
$$\sigma_{eq} = \sqrt{\left(\frac{F}{A}\right)^2 + 4\left(\frac{T}{W_\rho}\right)^2} \le [\sigma] \qquad \textbf{(4)}$$

Applying the energy theory of strength, we obtain:

$$\sigma_{eq} = \sqrt{\left(\frac{F}{A}\right)^2 + 3\left(\frac{T}{W_\rho}\right)^2} \le [\sigma] \qquad \textbf{(5)}$$

The building of diagrams of twisting and bending moments

Let us consider the shaft:



(Fig. 1)

The building of diagrams of twisting and bending moments

The numerical dates for a given shaft:

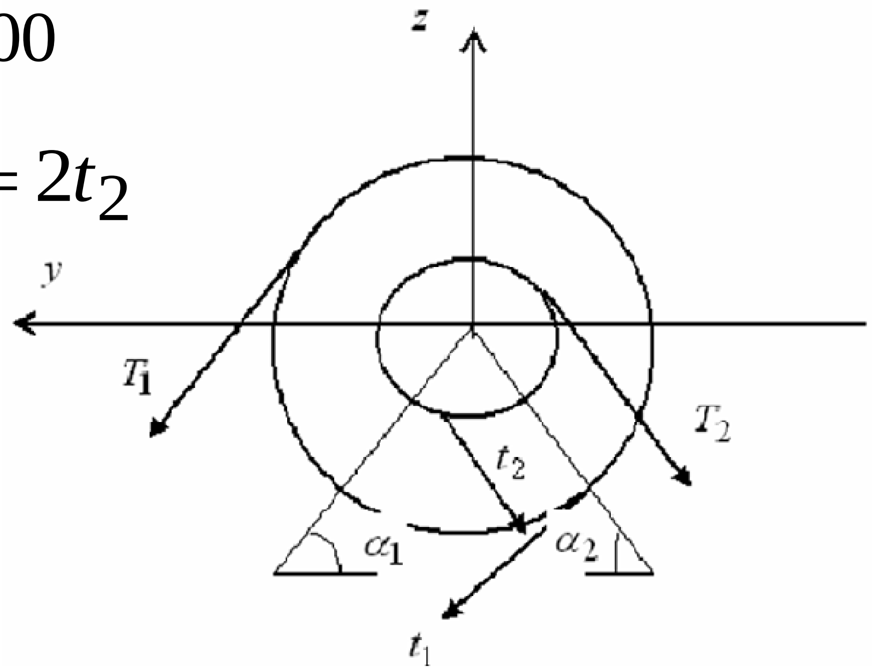
$D_1 = 0{,}9$ m $\quad D_2 = 0{,}3$ m $\quad n = 400$

$N = 50$ **kWt** $\quad T_1 = 2t_1 \quad T_2 = 2t_2$

$\alpha_1 = 45^0 \quad \alpha_2 = 45^0$

$a = 1{,}2$ m $\quad b = c = 0{,}5$ m

$[\sigma] = 70$ **kPa**



(Fig. 2)

The building of diagrams of twisting and bending moments

Let us determine the moments attached to the pulleys:

$$\omega = \frac{2\pi \cdot n}{60} = \frac{2 \cdot 3{,}14 \cdot 400}{60} \approx 41{,}9 \text{ rad/s}$$

Then the torque applied to the first pulley is equal:

$$M_1 = \frac{N}{\omega} = \frac{50000}{41{,}9} = 1193 \text{ Nm}$$

$$M_2 = \frac{N}{2\omega} = \frac{50000}{2 \cdot 41{,}9} = 596{,}5 \text{ Nm}$$

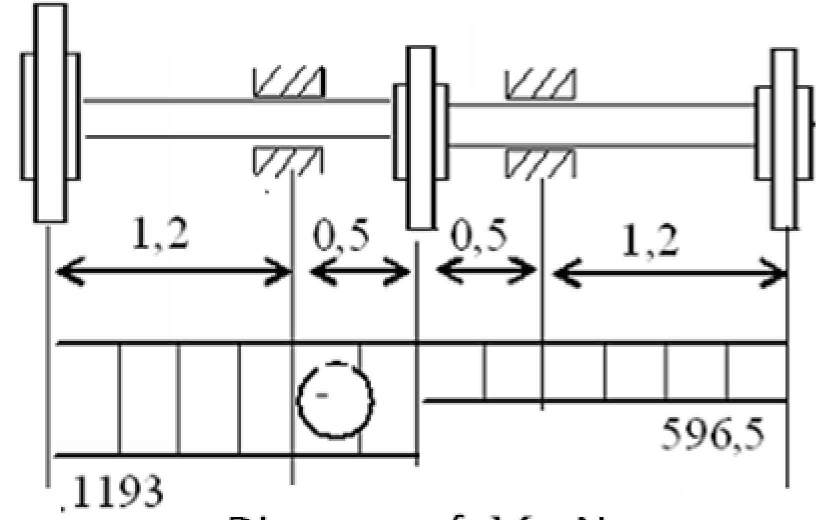


Diagram of M_k Nm

(Fig. 3)

The building of diagrams of twisting and bending moments

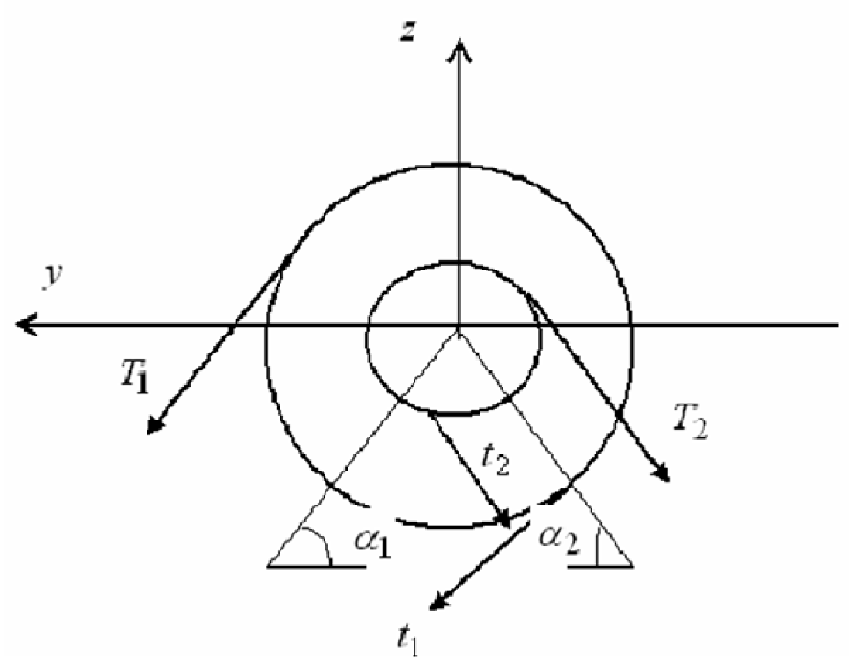
Determine the horizontal and vertical components of forces P1 and P2 according to the scheme of their location (Fig. 2):

$$H_1 = R_1 \cdot \cos\alpha_1 = 2813 \text{ N}$$

$$H_2 = -R_2 \cdot \cos\alpha_2 = -4221 \text{ N}$$

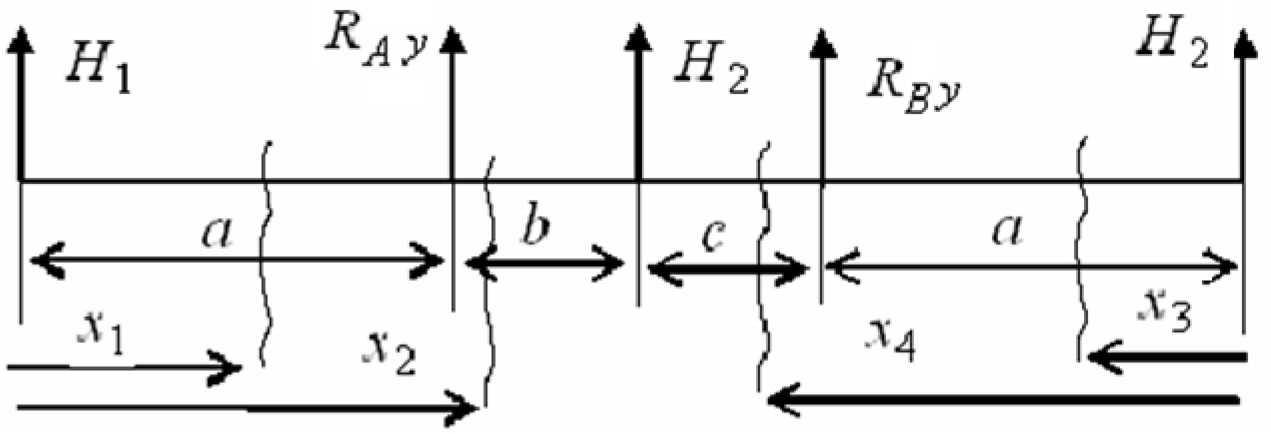
$$V_1 = -R_1 \cdot \sin\alpha_1 = -2813 \text{ N}$$

$$V_2 = -R_2 \cdot \sin\alpha_2 = -4221 \text{ N}$$



The building of diagrams of twisting and bending moments

Let us consider the stress state of shaft, which is loaded by horizontal forces (Fig. 4):



(Fig. 4)

The building of diagrams of twisting and bending moments

To find the reaction of shaft supports

$$\sum M_{B_i} = 0$$

$$R_{Ay} = \frac{-H_1(a+b+c) + H_2(a-c)}{b+c} = -9143{,}3 \text{ N}$$

$$\sum M_{A_i} = 0$$

$$R_{By} = \frac{H_1 a - H_2(a+2b+c)}{b+c} = 14772{,}3 \text{ N}$$

The building of diagrams of twisting and bending moments

To find the reaction of shaft supports

$$\sum M_{B_i} = 0$$

$$R_{Ay} = \frac{-H_1(a+b+c) + H_2(a-c)}{b+c} = -9143{,}3 \text{ N}$$

$$\sum M_{A_i} = 0$$

$$R_{By} = \frac{H_1 a - H_2(a+2b+c)}{b+c} = 14772{,}3 \text{ N}$$

Verification of the reactions of supports, which were determinated:

$$\sum F_{yi} = 0 \qquad H_1 + 2H_2 + R_{Ay} + R_{By} =$$

$$= 2813 - 2 \cdot 4221 - 9143{,}3 + 14772{,}3 = -5629 + 5629 = 0$$

The building of diagrams of twisting and bending moments

To construct the diagram of the bending moment My

I portion $0 \le x_1 \le 1{,}2$ $M_{y_1} = H_1 \cdot x_1$ $M(0) = 0$ $M(1{,}2) = 3376$ Nm

II portion $1{,}2$ m $\le x_2 \le 1{,}7$ m $M_{y_2} = H_1 \cdot x_2 + R_A^y \cdot (x_2 - 1{,}2)$

$M(1{,}7) = 211$ Nm

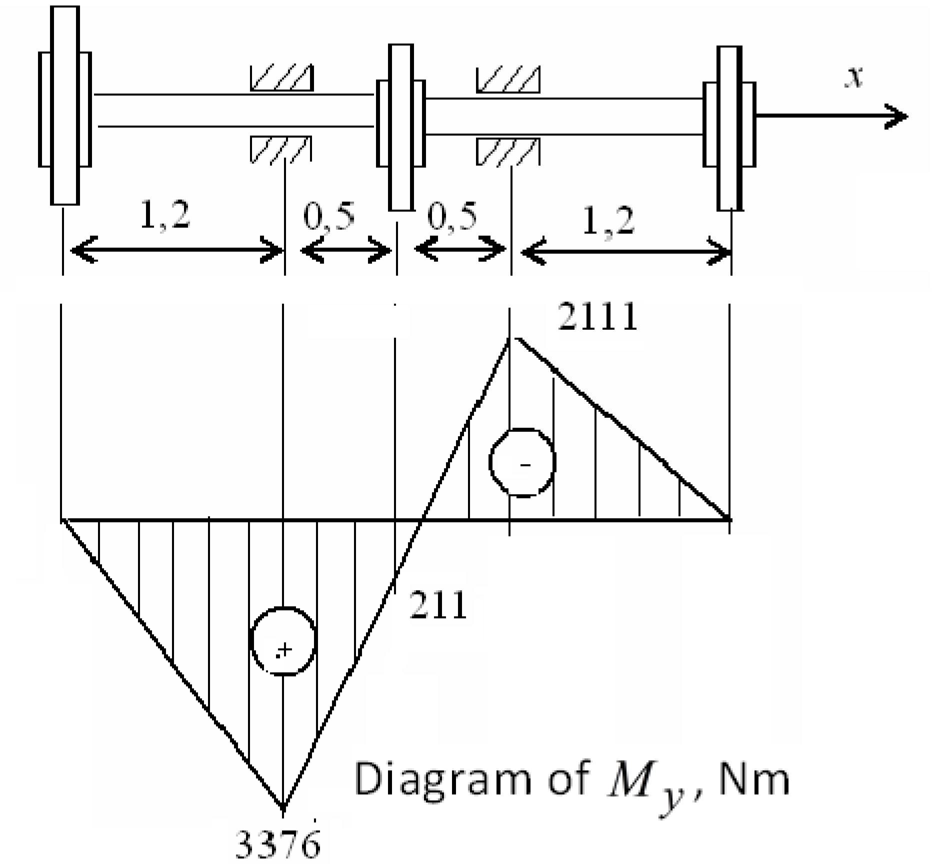
III portion $0 \le x_3 \le 1{,}2$ $M_{y_3} = H_2 \cdot x_3$

$M(0) = 0$ $M(1{,}2) = -2111$ Nm

IV portion $1{,}2$ m $\le x_4 \le 1{,}7$ m $M_{y_4} = H_2 \cdot x_4 + R_B^y \cdot (x_4 - 1{,}2)$

$M(1{,}7) = 211$ Nm

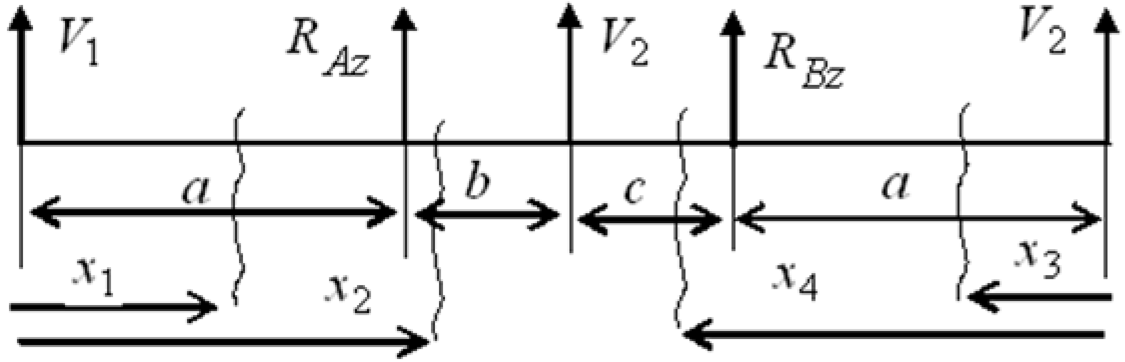
The building of diagrams of twisting and bending moments



(Fig. 5)

The building of diagrams of twisting and bending moments

Let us consider the stress state of shaft, which is loaded by vertical forces (Fig. 6):



(Fig. 6)

The building of diagrams of twisting and bending moments

To find the reaction of shaft supports

$$\sum M_{B_i} = 0$$

$$R_{Az} = \frac{-V_1(a+b+c)+V_2(a-c)}{b+c} = 3233{,}9$$ N

$$\sum M_{A_i} = 0$$

$$R_{Az} = \frac{-V_1(a+b+c)+V_2(a-c)}{b+c} = 3233{,}9$$ N

Verification of the reactions of supports, which were determinated:

$$\sum F_{zi} = 0$$

$$V_1 + 2V_2 + R_{Az} + R_{Bz} =$$

$$= -2813 - 2 \cdot 4221 + 3233{,}9 + 8021{,}1 = -11255 + 11255 = 0$$

The building of diagrams of twisting and bending moments

To construct the diagram of the bending moment Mz

I portion $0 \le x_1 \le 1{,}2$ $M_{z_1} = V_1 \cdot x_1$ $M(0) = 0$ $M(1{,}2) = -3376$ Nm

II portion $1{,}2$ m $\le x_2 \le 1{,}7$ m $M_{z_2} = V_1 \cdot x_2 + R_A^z \cdot (x_2 - 1{,}2)$

$M(1{,}7) = -3165$ Nm

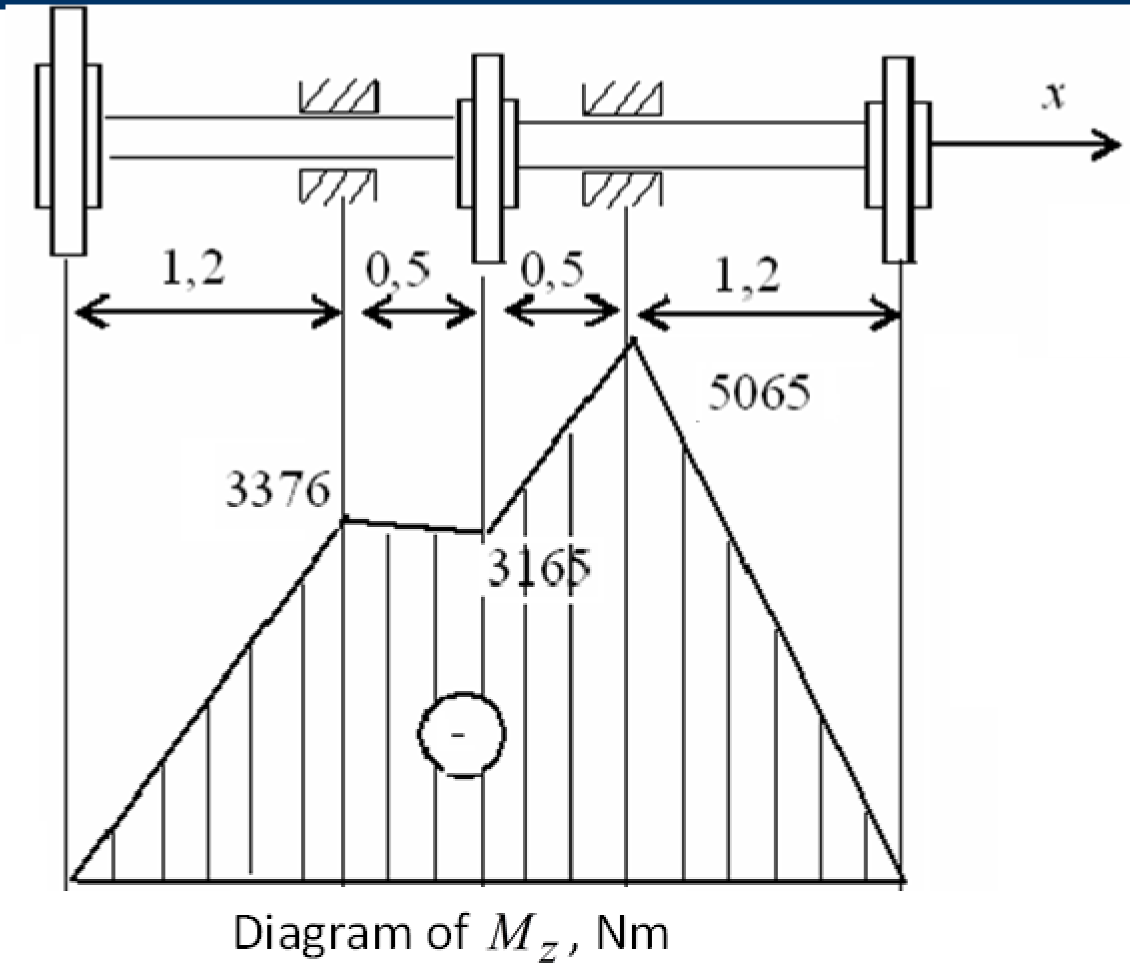
III portion $0 \le x_3 \le 1{,}2$ $M_{z_3} = V_2 \cdot x_3$

$M(0) = 0$ $M(1{,}2) = -5065$ Nm

IV portion $1{,}2$ m $\le x_4 \le 1{,}7$ m $M_{z_4} = V_2 \cdot x_4 + R_B^z \cdot (x_4 - 1{,}2)$

$M(1{,}7) = -3165$ Nm

The building of diagrams of twisting and bending moments



(Fig. 7)

The building of diagrams of twisting and bending moments

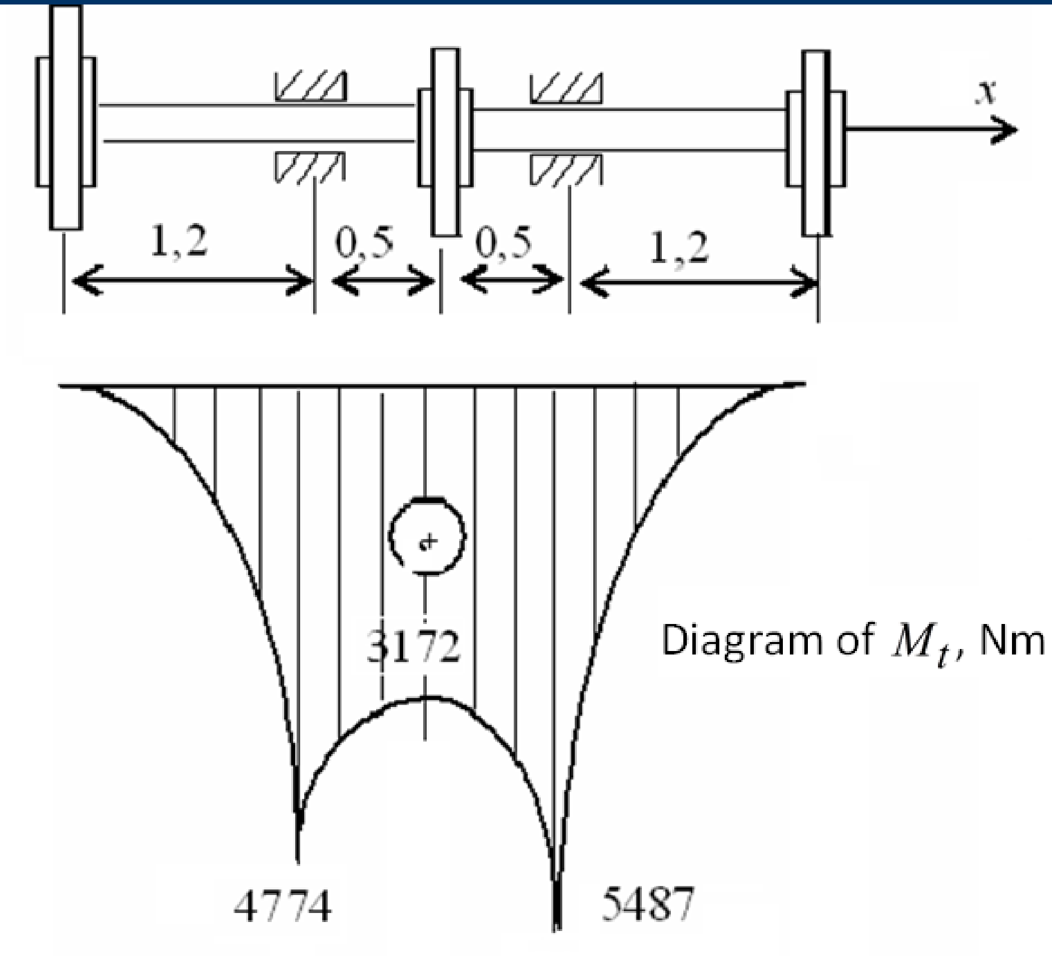
To construct a diagram of the total bending moment

$$M_{t_1} = \sqrt{M_{y_1}^2 + M_{z_1}^2} = \sqrt{3376^2 + (-3376)^2} = 4774 \text{ Nm}$$

$$M_{t_2} = \sqrt{M_{y_2}^2 + M_{z_2}^2} = \sqrt{211^2 + (-3165)^2} = 3172 \text{ Nm}$$

$$M_{t_3} = \sqrt{M_{y_3}^2 + M_{z_3}^2} = \sqrt{(-2111)^2 + (-5065)^2} = 5487 \text{ Nm}$$

The building of diagrams of twisting and bending moments



(Fig. 8)

Picking up of shafte diameter from strengt condition

Determine the calculated moment by the third strength theory:

$$M_{III} = \sqrt{M_{t_i}^2 + M_{\kappa_i}^2}$$

or:

$$M_{III} = \sqrt{5487^2 + 596{,}5^2} = 5519{,}3 \text{ Nm}$$

The diameter of the shaft is determined from the condition of strength:

$$d \geq \sqrt[3]{\frac{32 \cdot M_{III}}{\pi \cdot [\sigma]}} = \sqrt[3]{\frac{32 \cdot 5519{,}3}{3{,}14 \cdot 70 \cdot 10^3}} \approx 0{,}93 \text{ m}$$

We accept a diameter equal to 100 mm.