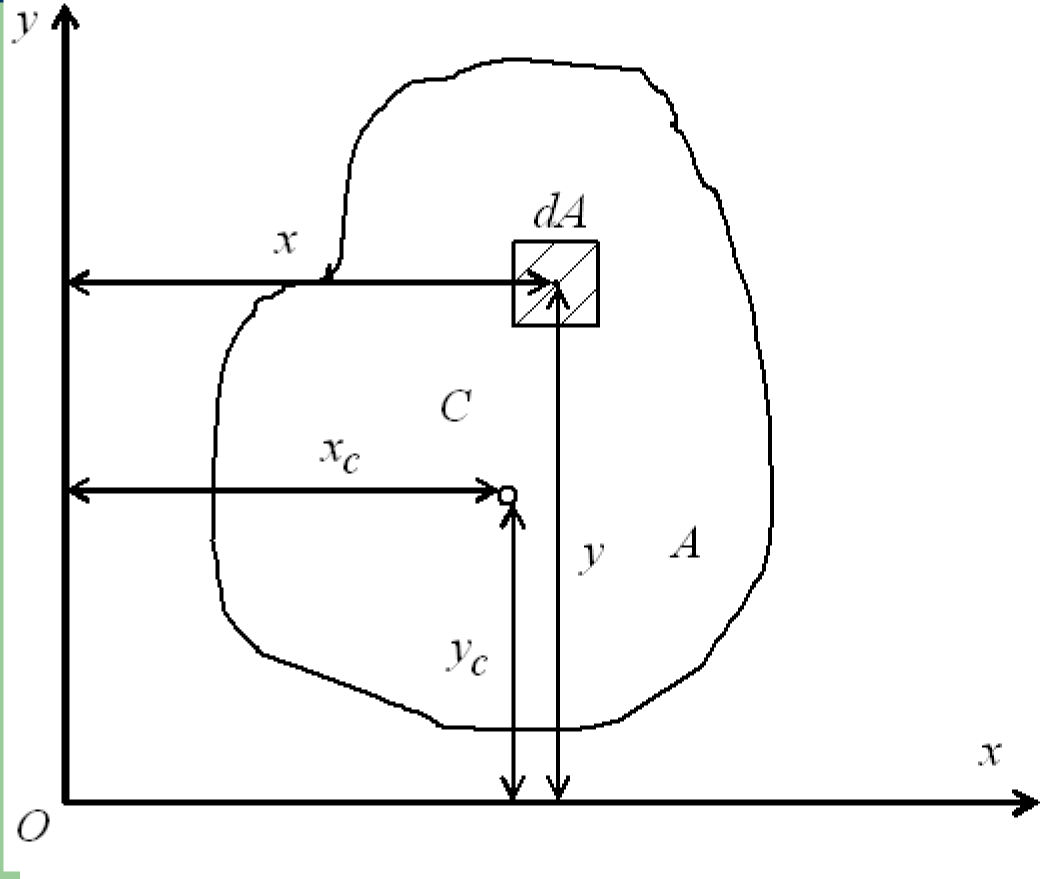
Lecture 5.
FIRST MOMENT OF AREA

Assos. Prof. Kutsenko

Plan of lecture

- **1. Definition of first moment of area**
- **2. Centroid of an area**

Definition of first moment of area



$$dS_x = ydA$$

(1)

$$dS_y = xdA$$

Fig.1

Definition of first moment of area

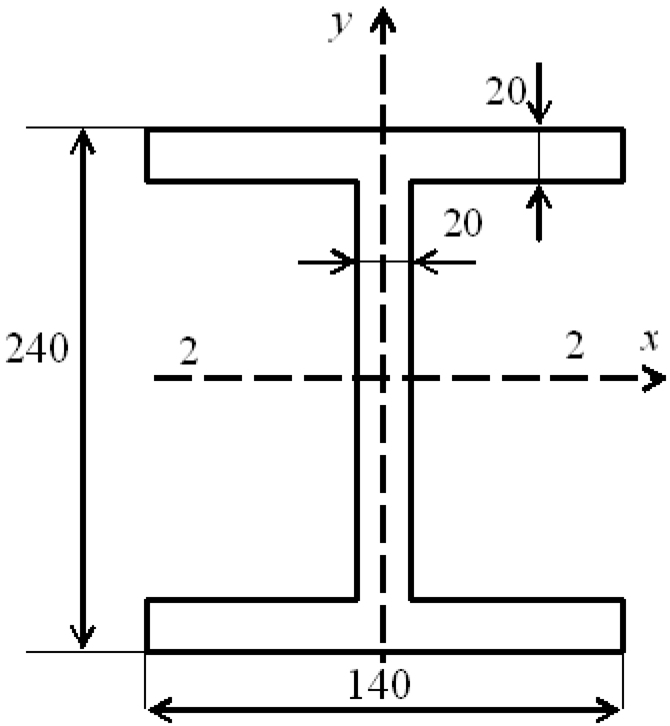


Fig.2

Example 1

Define the static moments of parts of area a cross-section of beam which are located higher line 2-2 in relation to central main axis

Definition of first moment of area

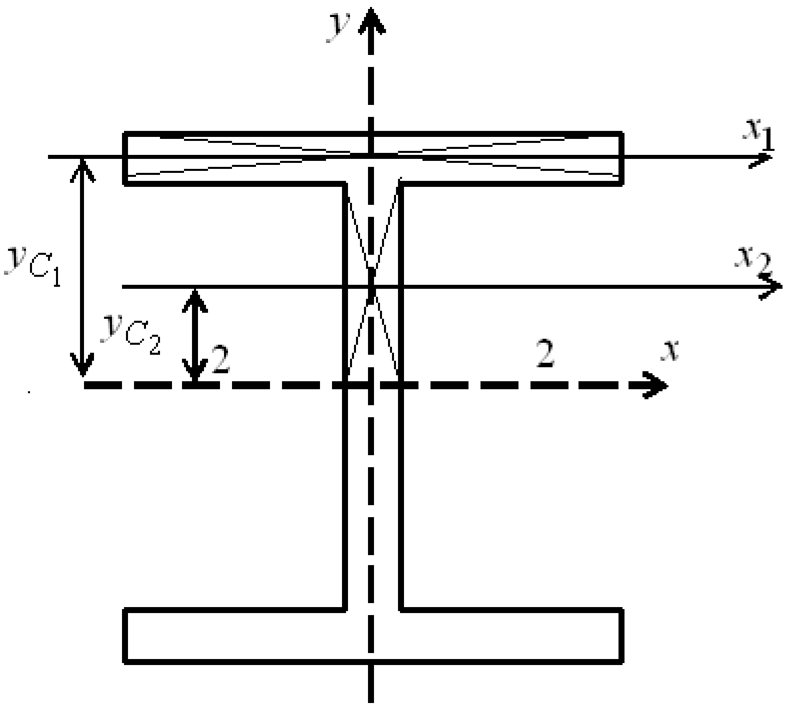


Fig.3

Example 1

Breaking up this section on three rectangles with according areas:

$$A_1 = 2 \times 14 = 28 \quad \textbf{cm}^2$$

$$A_2 = 2 \times 10 = 20 \quad \textbf{cm}^2$$

Definition of first moment of area

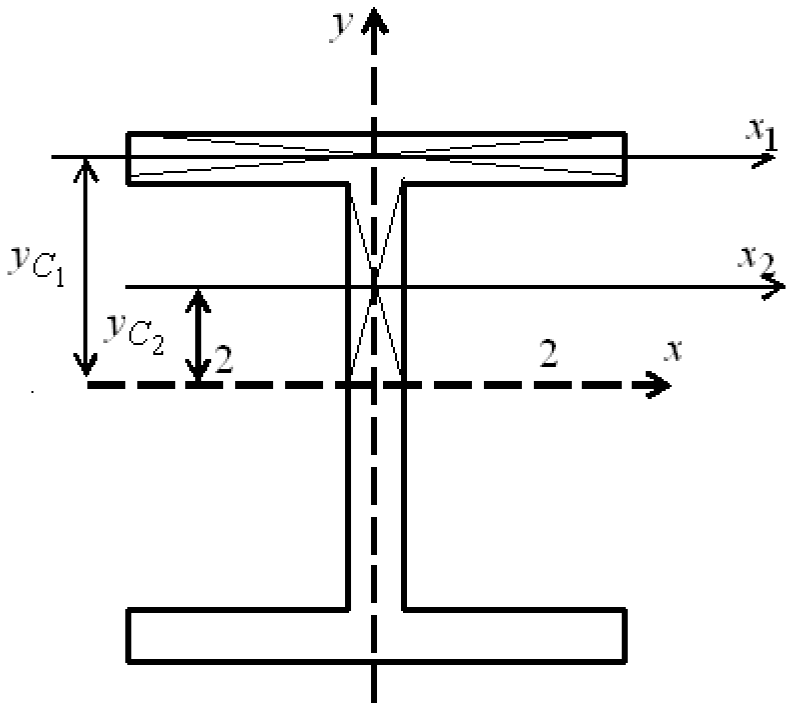


Fig.3

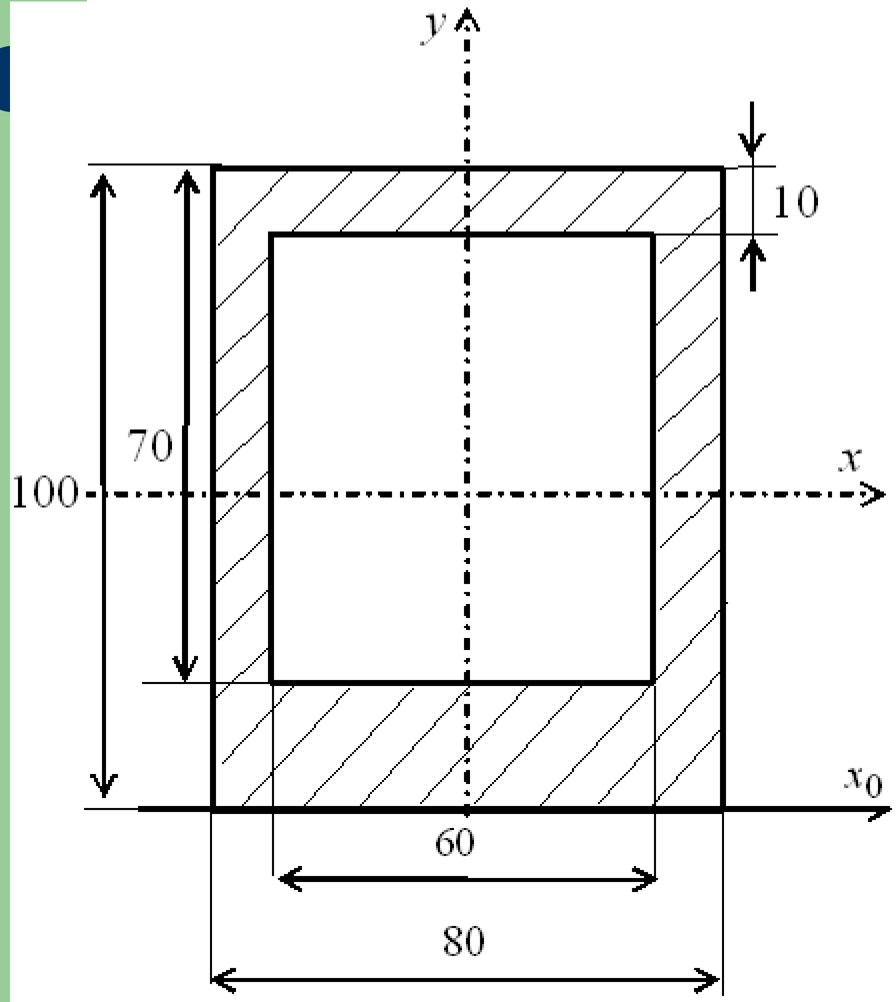
Example 1

The first moment of area about an axis which considers straight-in 2 - 2 will be written down so:

$$S_x^{2-2} = A_1 \cdot y_{C_1} + A_2 y_{C_2} =$$

$$= 28 \cdot 11 + 20 \cdot 5 = 408 \text{ cm}^3$$

Definition of first moment of area

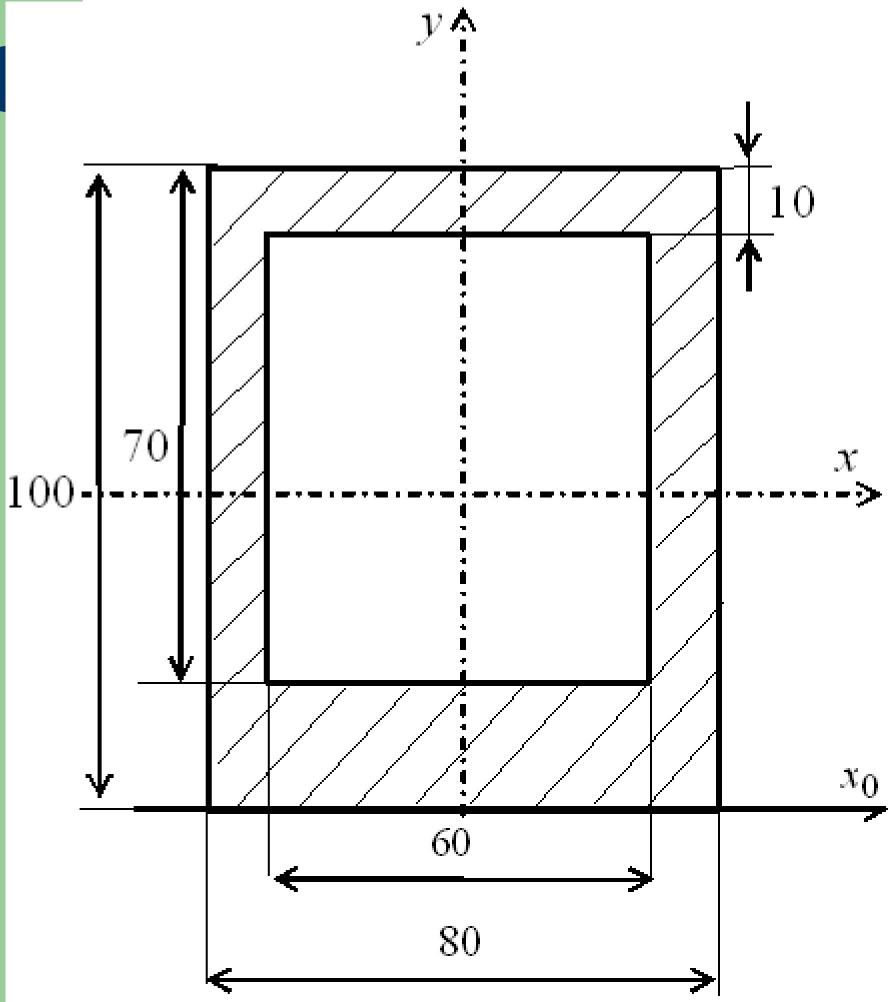


Example 2

Determine the first moment of area of a section in relation to an axis x_0

Fig.4

Definition of first moment of area



Example 2

Calculate an area of each of rectangles:

$$A_1 = 10 \cdot 8 = 80 \text{ cm}^2$$

$$A_2 = 7 \cdot 6 = 42 \text{ cm}^2$$

Fig.4

Definition of first moment of area

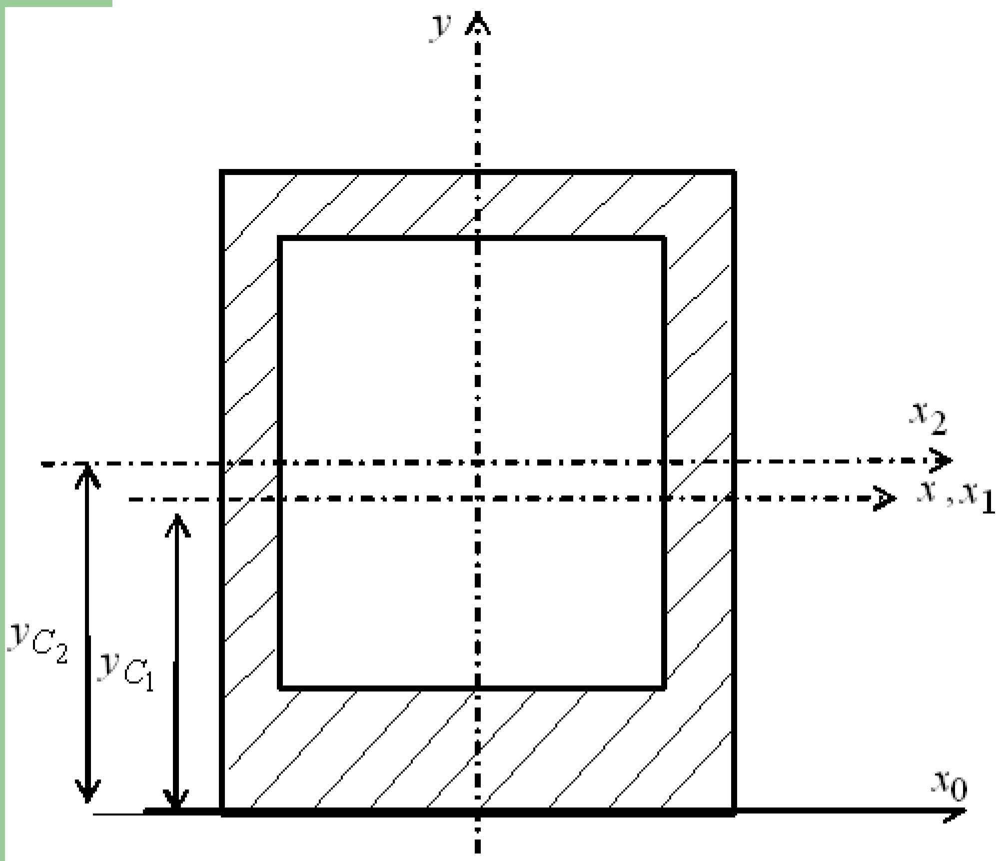


Fig. 5

Example 2

The distances from axes x_1 and x_2 accordingly will be equal:

$$y_{C_1} = 5 \text{ cm}$$

$$y_{C_2} = 2 + 3{,}5 = 5{,}5 \text{ cm}$$

Definition of first moment of area

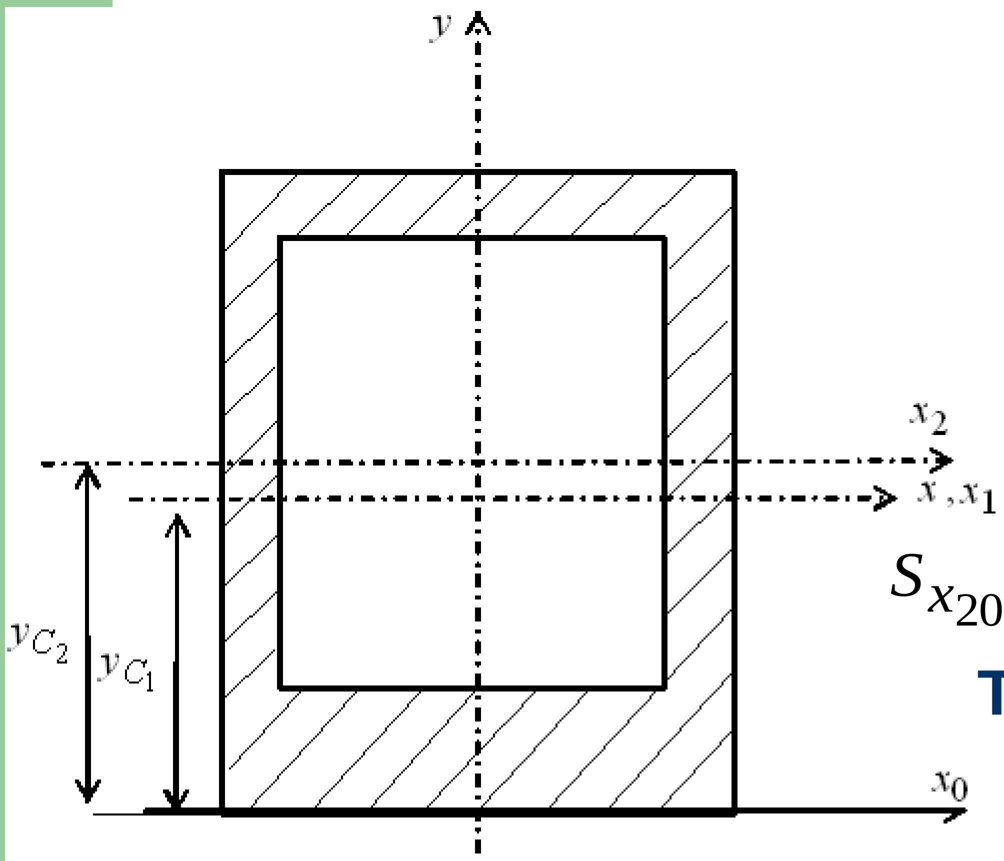


Fig. 5

Example 2

The first moments of each of rectangles, we get:

$$S_{x_{10}} = A_1 \cdot y_{C_1} = 80 \cdot 5 = 400 \ \textbf{cm}^3$$

$$S_{x_{20}} = A_2 \cdot y_{C_2} = 42 \cdot 5{,}5 = 231 \ \textbf{cm}^3$$

Then $S_{x_0} = S_{x_{10}} - S_{x_{20}} =$

$$= 400 - 231 = 169 \ \textbf{cm}^3$$

Centroid of an area

The centroid of an area is the point at which the area might be considered to be concentrated and still leaves unchanged the first moment of the area about any axis

The centroid of an area is defined by the equations:

$$y_C = \frac{S_x}{A} = \frac{\int y dA}{A} \quad ; \qquad x_C = \frac{S_y}{A} = \frac{\int x dA}{A} \qquad \textbf{(2)}$$

Centroid of an area

For a plane area, which is composed of n subareas A_i, each of whose centroidal coordinates x_C and y_C are known, the integral is replaced by a summing:

$$x_C = \frac{S_y}{A} = \frac{\sum_{i=1}^{n} A_i x_{C_i}}{\sum_{i=1}^{n} A_i} \quad ; \quad y_C = \frac{S_x}{A} = \frac{\sum_{i=1}^{n} A_i y_{C_i}}{\sum_{i=1}^{n} A_i} \qquad \textbf{(3)}$$

Centroid of an area

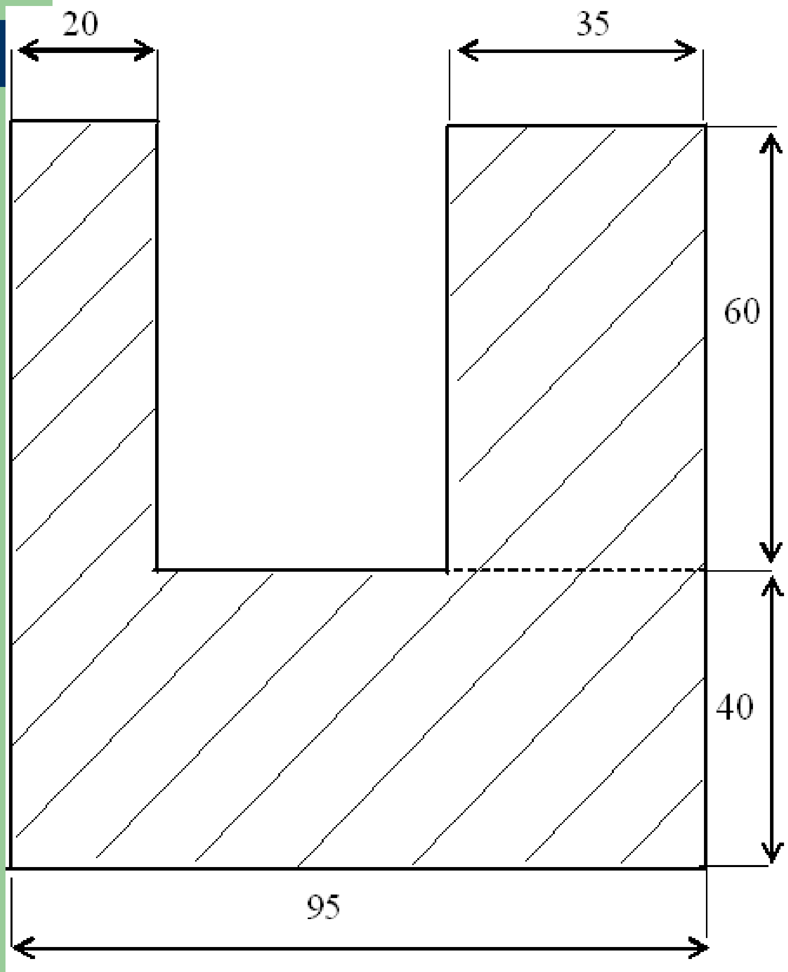
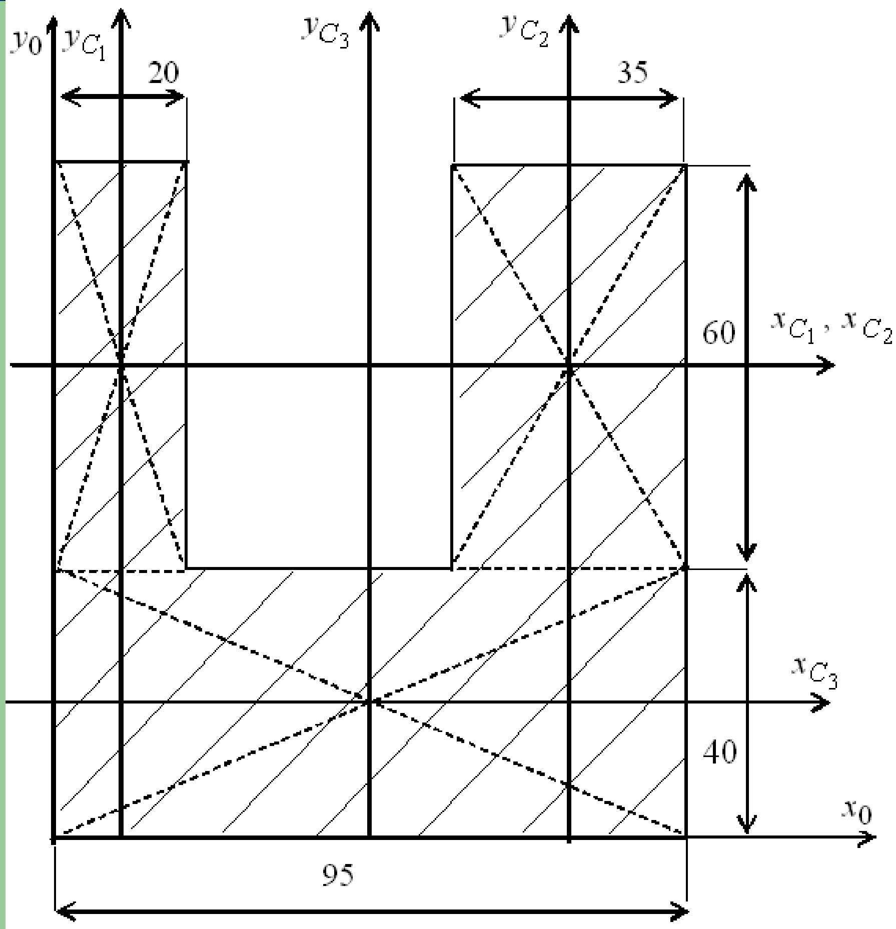


Fig. 6

Example 3

Determine position of centeroid of cross-section and conduct central axes x_C and y_C

Centroid of an area



Example 3

The areas of the corresponding rectangles there will be equals:

$$A_1 = 2 \times 6 = 12 \quad \text{cm}^2$$

$$A_2 = 3{,}5 \times 6 = 21 \quad \text{cm}^2$$

$$A_3 = 9{,}5 \times 4 = 38 \quad \text{cm}^2$$

Fig. 6

Centroid of an area

Conduct central axes for each of three rectangles and define distances from them x_{Ci} and y_{Ci} to the initial auxiliary axes x_0 and y_0:

1. $x_{C_1} = \frac{2}{2} = 1$ **cm** $y_{C_1} = 3 + 4 = 7$ **cm** $A_1 = 12$ **cm²**

2. $x_{C_2} = 9{,}5 - \frac{3{,}5}{2} = 7{,}75$ **cm** $y_{C_2} = 3 + 4 = 7$

$A_2 = 21$ **cm²**

3. $x_{C_3} = \frac{9{,}5}{2} = 4{,}75$ **cm** $y_{C_3} = \frac{4}{2} = 2$ **cm** $A_3 = 38$ **cm²**

Centroid of an area

Then in obedience to formula (3), we get:

$$x_C = \frac{S_y}{A} = \frac{\sum_{i=1}^{n} A_i x_{C_i}}{\sum_{i=1}^{n} A_i} = \frac{x_{C_1} \cdot A_1 + x_{C_2} \cdot A_2 + x_{C_3} \cdot A_3}{A_1 + A_2 + A_3} =$$

$$= \frac{1 \cdot 12 + 7,75 \cdot 21 + 4,75 \cdot 38}{12 + 21 + 38} = \frac{355,25}{71} \approx 5$$ **cm**

Centroid of an area

$$y_C = \frac{S_x}{A} = \frac{\sum_{i=1}^{n} A_i y_{C_i}}{\sum_{i=1}^{n} A_i} = \frac{y_{C_1} \cdot A_1 + y_{C_2} \cdot A_2 + y_{C_3} \cdot A_3}{A_1 + A_2 + A_3} =$$

$$= \frac{7 \cdot 12 + 7 \cdot 21 + 2 \cdot 38}{12 + 21 + 38} = \frac{307}{71} \approx 4{,}32$$ **cm**

Centroid of an area

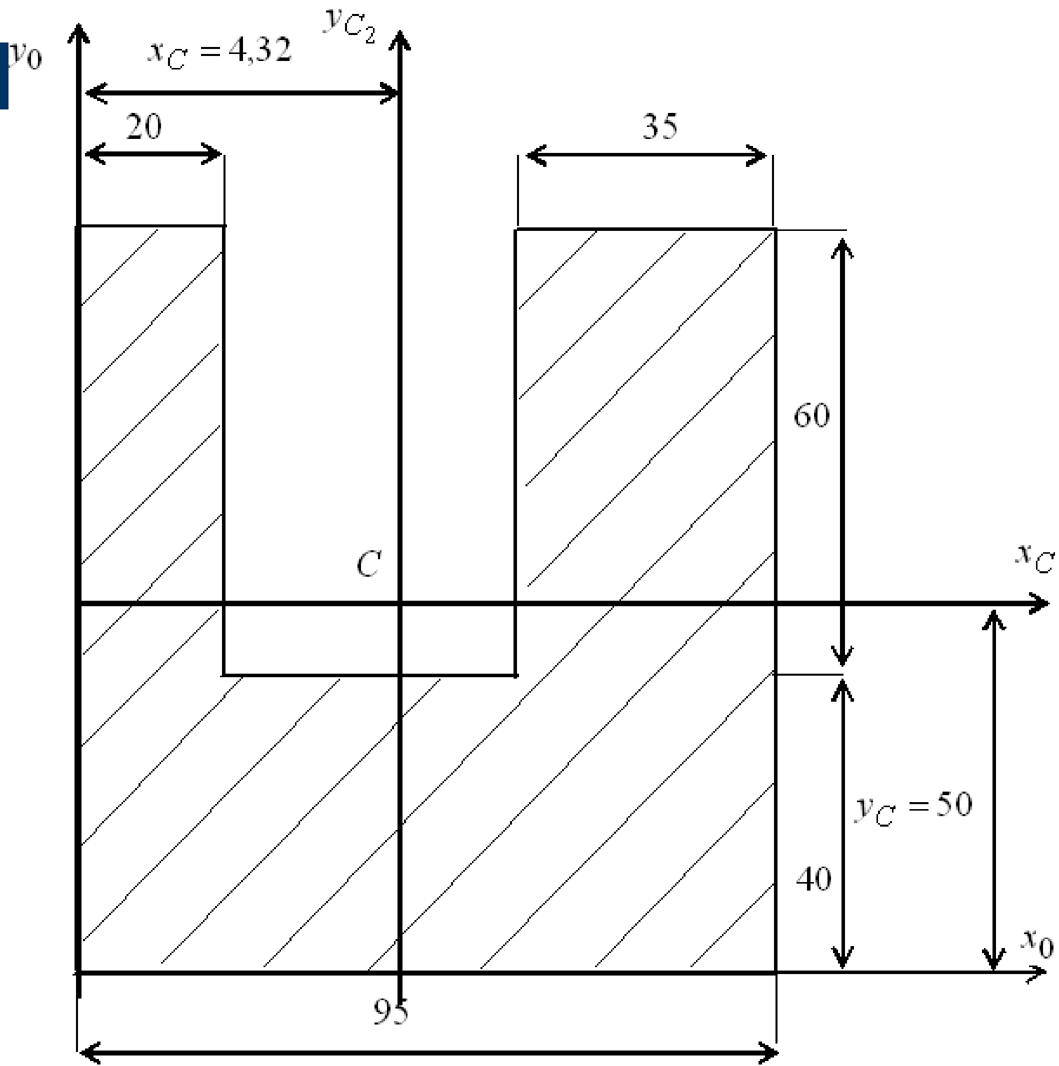


Fig. 7

Thank you!

Good bye!