Lecture 6 - 7.
MOMENTS OF INERTIA

Assos. Prof. Kutsenko

Plan of lecture

- 1. Kinds of moment of inertia and their definitions
- 2. The parallel - axis theorem for moment of inertia of a finite area
- 3. Principal moments of inertia

Definition of moment of inertia

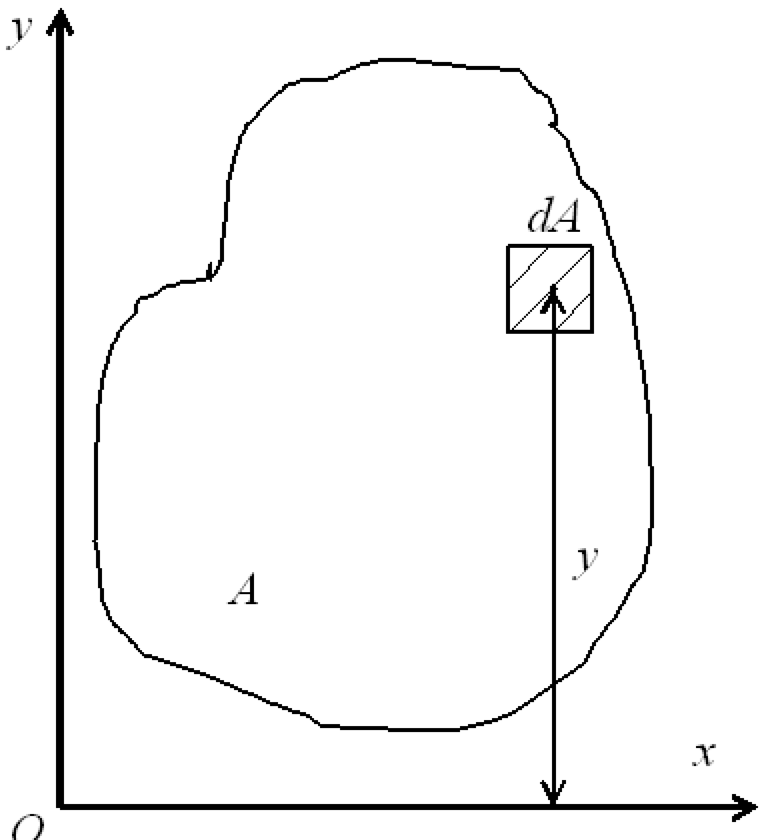

Fig.1

If the moment of inertia of the finite area about the x - axis is denoted by I_x, then we get:

$$I_x = \int y^2 dA \quad \textbf{(1)}$$

Definition of moment of inertia

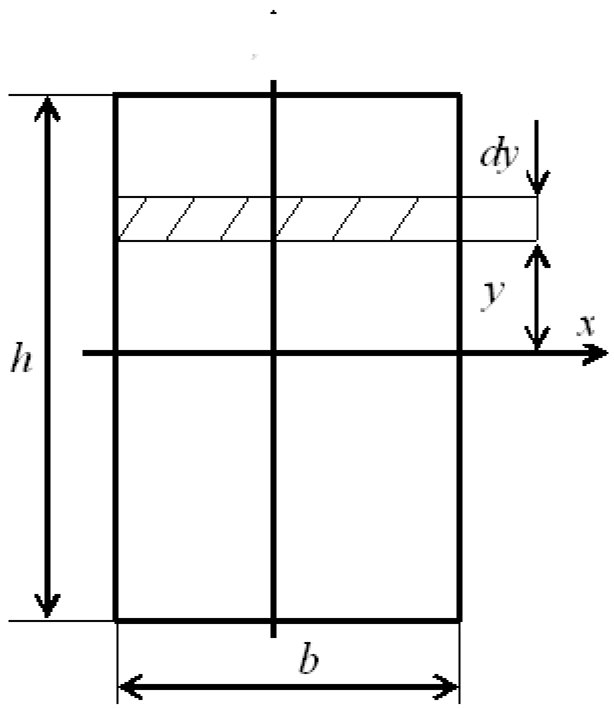

Fig. 2

Example 1

For given rectangle determine the moment of inertia about an x-axis, which pass through the its centroid

Definition of moment of inertia

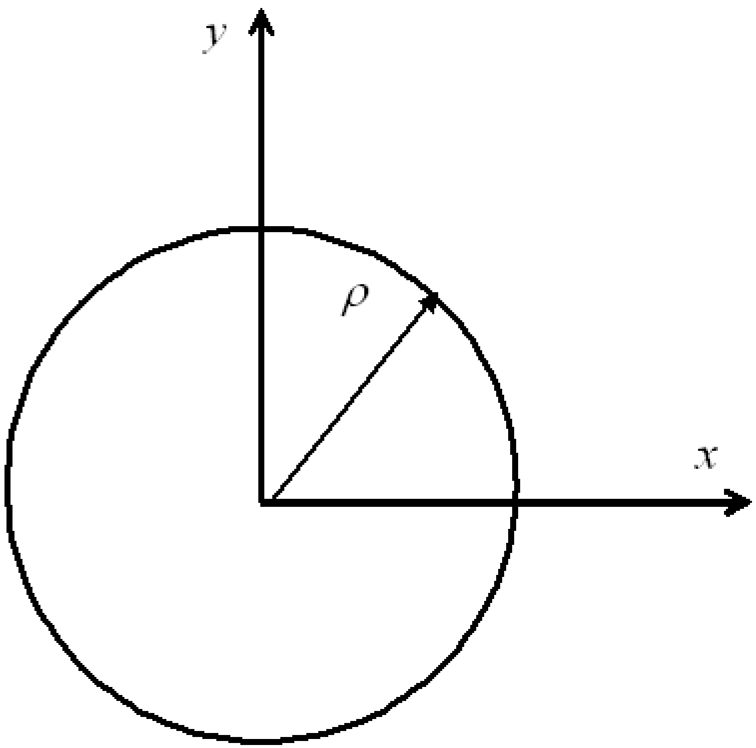

Fig. 3

By definition, the polar moment of inertia is given by the integral:

$$I_\rho = \int_A \rho^2 dA \quad \textbf{(3)}$$

Definition of moment of inertia

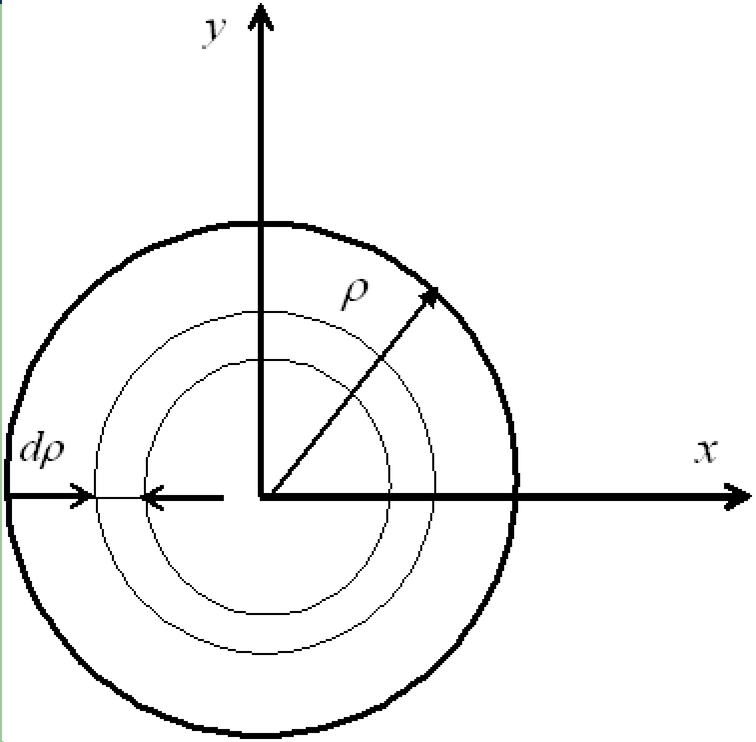

Fig. 4

Example 2

For given circle determine the polar moment of inertia

Definition of moment of inertia

The product of inertia of a finite area with respect to the x- and y- axes in the plane of the area is given by the summation of the products of inertia about those same axes of all elements of area contained within the finite area

$$I_{xy} = \int xy\,dA \qquad (5)$$

Definition of moment of inertia

For a plane area composed of n subareas A_i, each of whose moment of inertia is known about the x and y-axes, the integral is replaced by a summation:

$$I_x = \sum_{i=1}^{n} I_{x_i}$$

(6)

$$I_y = \sum_{i=1}^{n} I_{y_i}$$

The parallel - axis theorem for moment of inertia of a finite area

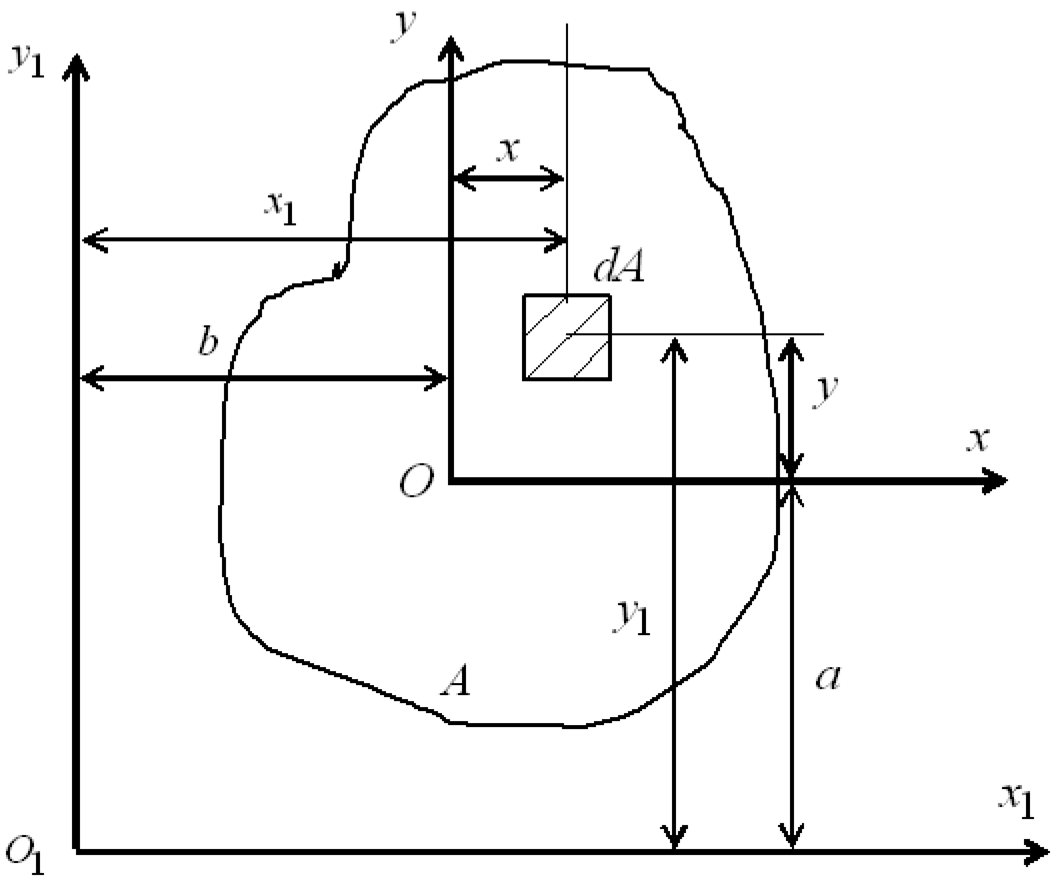

Fig. 5

Let us consider the plane area shown in Fig. 5

The moments I_x and I_y of inertia about the axes through the centroid are known

The parallel - axis theorem for moment of inertia of a finite area

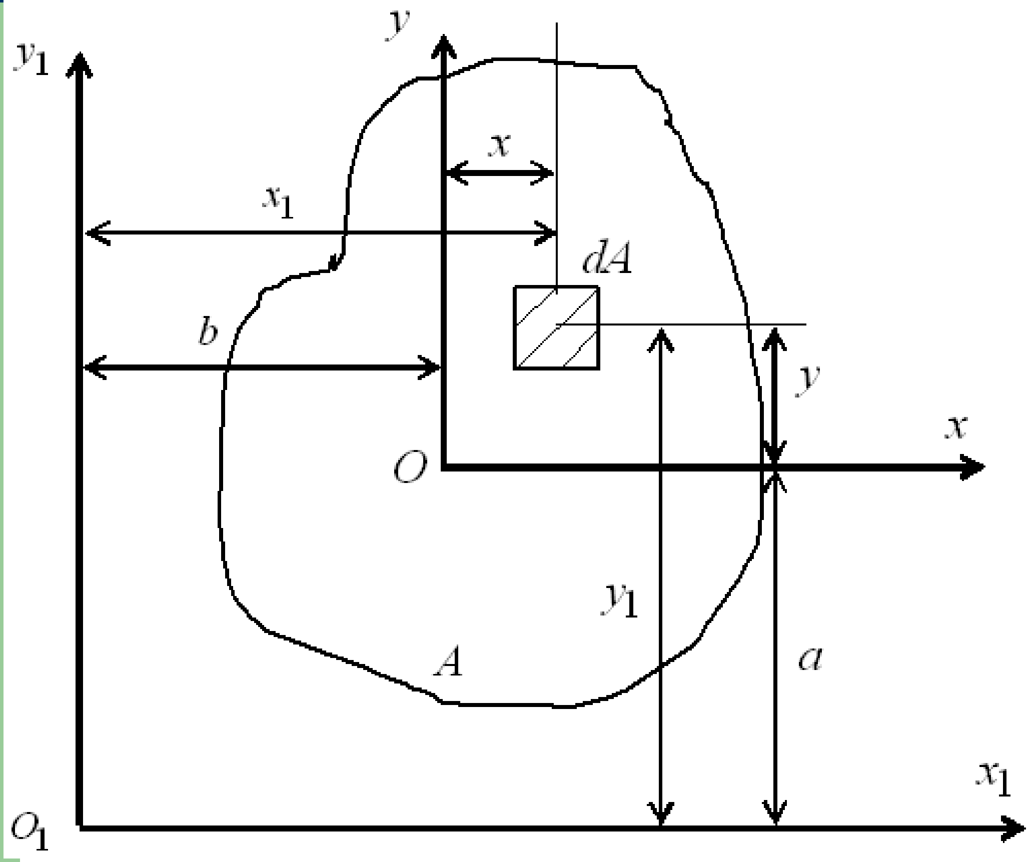

Fig. 5

$$I_{x_1} = I_x + a^2 A$$

$$I_{y_1} = I_y + b^2 A \qquad \textbf{(7)}$$

$$I_{x_1 y_1} = I_{xy} + abA$$

The parallel - axis theorem for moment of inertia of a finite area

In general case for a plane area composed of n subareas A_i, each of whose moment of inertia is known about the x_i and y_i- axes, the integral is replaced by a summation:

$$I_{x_1} = \sum_{i=1}^{n} \left(I_{x_i} + a_i^2 A_i \right)$$

$$I_{y_1} = \sum_{i=1}^{n} \left(I_{y_i} + b_i^2 A_i \right) \qquad \textbf{(8)}$$

$$I_{x_1 y_1} = \sum_{i=1}^{n} \left(I_{x_i y_i} + a_i b_i A_i \right)$$

Principal moments of inertia

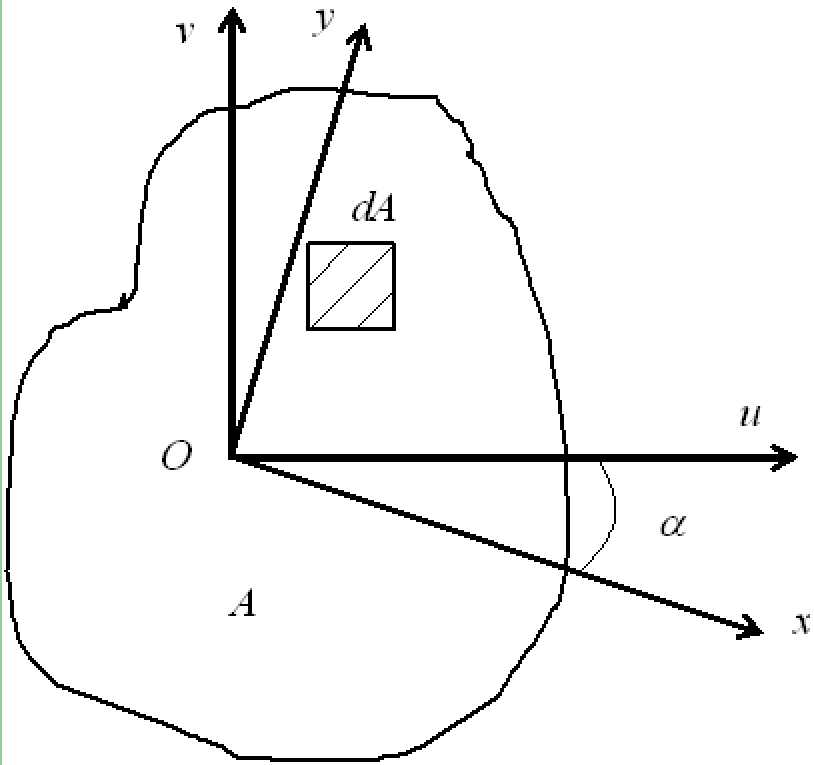

Fig. 6

Let us consider a plane area A and assume that I_x, I_y and I_{xy} are known. Determine the moments of inertia I_u and I_v as well as the product of inertia I_{uv} for the set of orthogonal axes u, v, oriented as shown in Fig. 6

Principal moments of inertia

These maximum and minimum values of moment of inertia are termed principal moments of inertia and are given by:

$$I_{\max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$I_{\min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2} \qquad \textbf{(10)}$$

Principal moments of inertia

The corresponding axes are termed principal axes:

$$tg2\alpha = \frac{2I_{xy}}{\left(I_y - I_x\right)} \qquad \textbf{(11)}$$

From (11) principal axes are defined

Thank you!

Good bye!