Lecture 9.
TORSION

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Plan of lecture

- 1. The basis expressions for torsion
- 2. The condition about the strength under torsion of shaft
- 3. The condition about the stiffness under torsion of shaft

The basis expressions for torsion

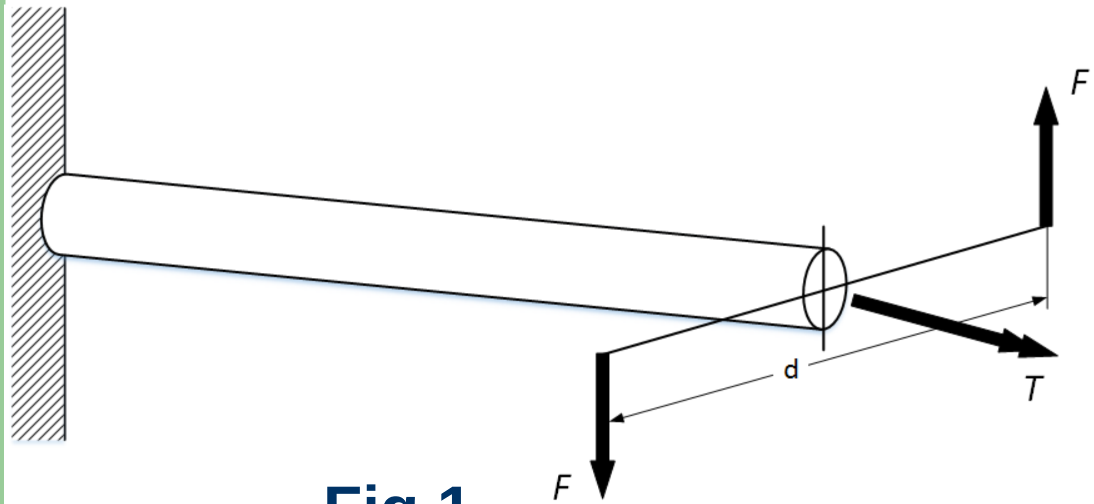
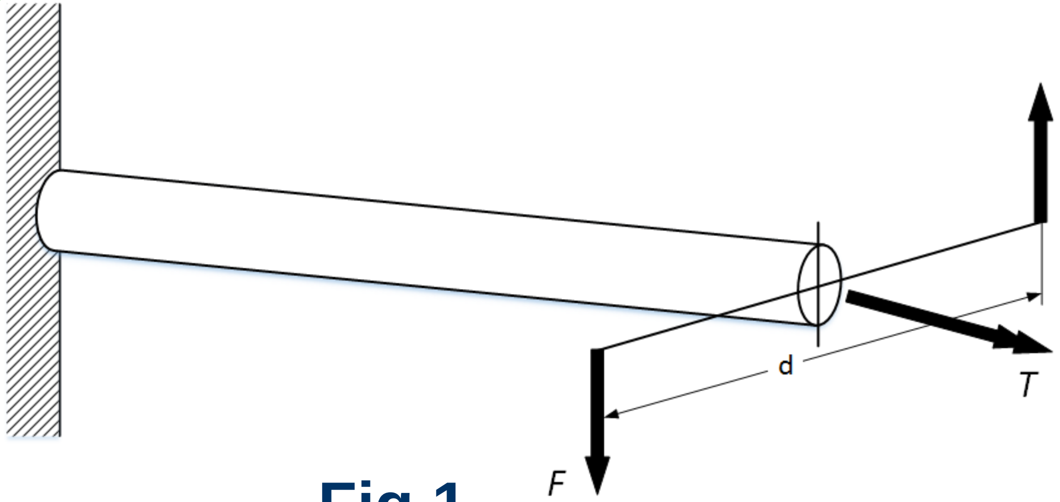


Fig.1

Consider a bar rigidly clamped at one and twisted at the other end by a torque (twisting moment)

$$T = F \cdot d \qquad (1)$$

The basis expressions for torsion



Such a bar is in torsion

Fig.1

This bar is called as a shaft

The basis expressions for torsion

Hooke's law :

$$\tau = G\gamma \qquad \textbf{(2)}$$

The tensional shearing stress is given by :

$$\tau = \frac{T \cdot \rho}{I_\rho} \qquad \textbf{(3)}$$

ρ ***- distance from the center of the shaft*** ;

T ***- twisting moment***

The basis expressions for torsion

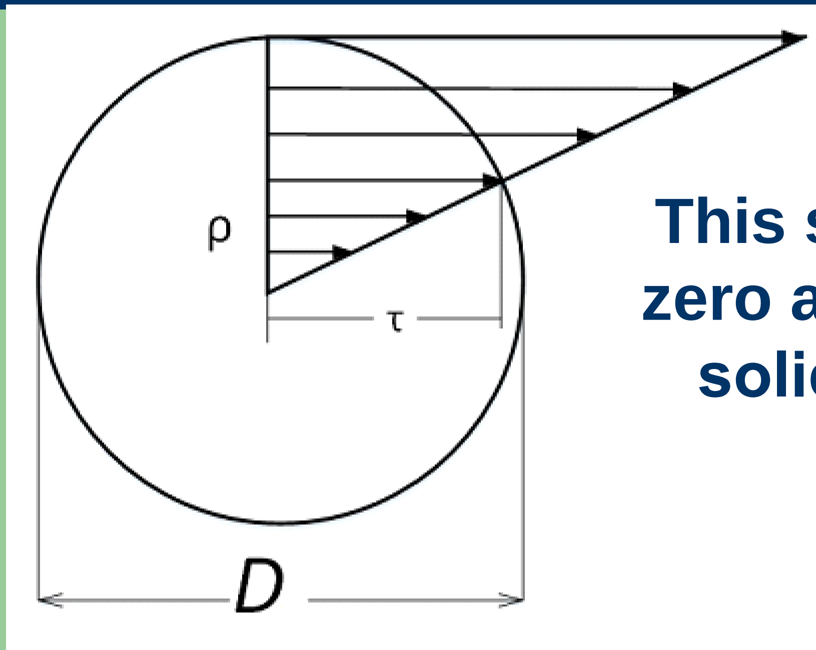


Fig. 2

This stress distribution varies from zero at the center of the shaft (if it is solid) to a maximum at the outer fibers

The basis expressions for torsion

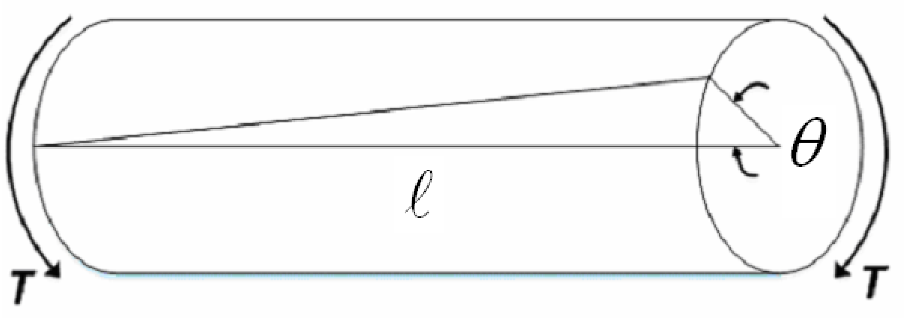


Fig. 3

The an expression for the angle of twist of a circular shaft as a function of the applied twisting moment :

$$\theta = \frac{T\ell}{GI_{\rho}} \qquad (4)$$

ℓ **- the shaft length**

The condition about the strength under torsion of shaft

From the expression (3) we can write the condition of strength for shaft:

$$\tau_{\max} = \frac{T_{\max}}{I_\rho} \rho_{\max} \le [\tau] \qquad \textbf{(5)}$$

I_ρ **- polar moment of inertia;**

From the expression (5) we can obtain the formula for diameter of shaft cross-section:

$$d \ge \sqrt[3]{\frac{16 T_{\max}}{\pi [\tau]}} \qquad \textbf{(6)}$$

The condition about the stiffness under torsion of shaft

From the expression (4) we can write the condition of stiffness for shaft:

$$\theta_{\max} = \frac{T_{\max}\,\ell}{GI_{\rho}} \leq [\theta] \qquad \textbf{(7)}$$

$c = \dfrac{GI_{\rho}}{\ell}$ **- stiffness of shaft;**

From the expression (7) we can obtain the formula for diameter of shaft cross-section:

$$d \geq \sqrt[4]{\frac{32T_{\max}}{\pi \cdot [\theta] \cdot G}} \qquad \textbf{(8)}$$

Thank you!

Good bye!