Lecture 25
COMBINED BENDING AND TENSION OR COMPRESSION

Plan

1. Classification of combined stresses.
2. Combined bending and tension or compression
3. Example of calculation of beam on bend and tension.

25.1 Classification of combined stresses

So far, we have studied the behaviour of deflection and force distribution in the bar, when it has only one kind of deformation. It were the simple kinds of deformation. In the analysis of simple types of deformations as a tension or a compression, a shear, a torsion, a bending, always considered one of the internal forces factors typical for a certain deformation. When studying the central compression in transverse sections of the bars, one main factor - normal force N; in the study of shear - shear force (transverse force) Q, in the study of twisting - the torque T and in the study of bending - bending moment M.

In actual practice, simultaneously several factors of internal forces acts in loaded elements of buildings. In this case, we have deal with a complex stresses. Thus, **a combined stresses is called such a stress state of building that can be replaced by a combination of the simplest kinds of deformations.**

In engineering calculations, combined stresses may be classified into the following four main types:

first type: **combined bending and tension or compression** is the case of stresses state, when in an arbitrary cross-section internal force factors are given to the normal force and to one or two bending moments are in the perpendicular planes;

second type: **a complex bending** is case of stresses state, when in each cross-section internal force factors are given two bending moments in the perpendicular planes;

third type: **the off-centre acting of force of tension or compression** is case of stresses state in which in any cross-section internal force factors are given to the normal force and to one or two bending moments in the perpendicular planes;

fourth type: **combined bending and torsion** is case of stresses state at which in arbitrary cross-section the internal forces are leaded to the twisting and bending moments.

Let us consider the basic relationships between stresses and deformation for each of the above stressed states.

On beginning

25.2 Combined bending and tension or compression

Consider the problem of determining the normal stresses in arbitrary cross-section of a straight beam of big stiffness (Fig. 25.1), when we have direct transverse bending by force F_2 and simultaneously tension by force F_1.

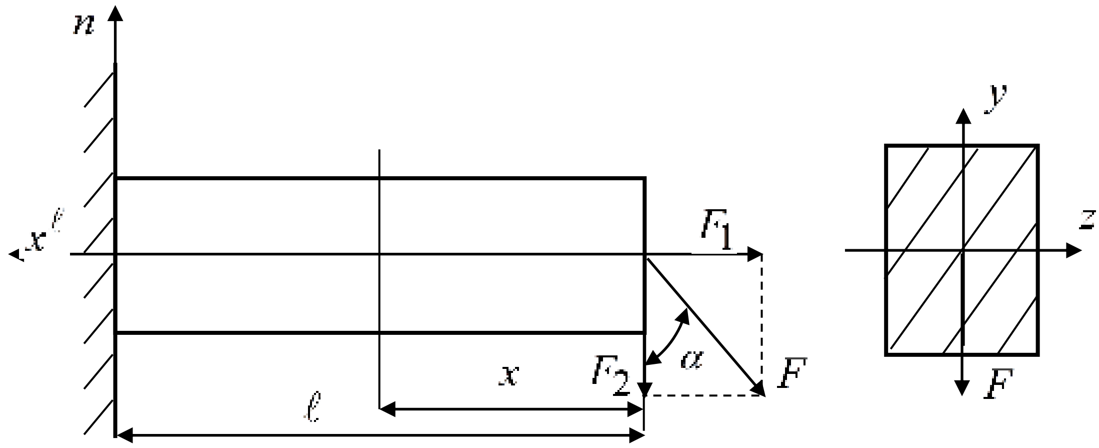

Fig. 26.1

This type of combined stresses is caused by the action of force F, which forms an angle α with the plane of the cross-section of the free end of the beam.

Since the beam has a constant cross-sectional shape, the value of normal stresses in arbitrary cross-section is determined by the formula:

$$\sigma_N = \frac{N}{A} \tag{25.1}$$

where N is the value of normal force in arbitrary cross-section of beam; A is the area of beam cross-section.

The stresses by bending moment which are caused by force are determined by the formula:

$$\sigma_y = \frac{M_x \cdot y}{I_z}. \tag{25.2}$$

Then the magnitude of total stresses, due to the principle of the independence acting of forces, at an arbitrary point of the cross-section of the beam, will determine from:

$$\sigma = \sigma_N + \sigma_y = \frac{N}{A} + \frac{M_x \cdot y}{I_z}. \qquad (25.3)$$

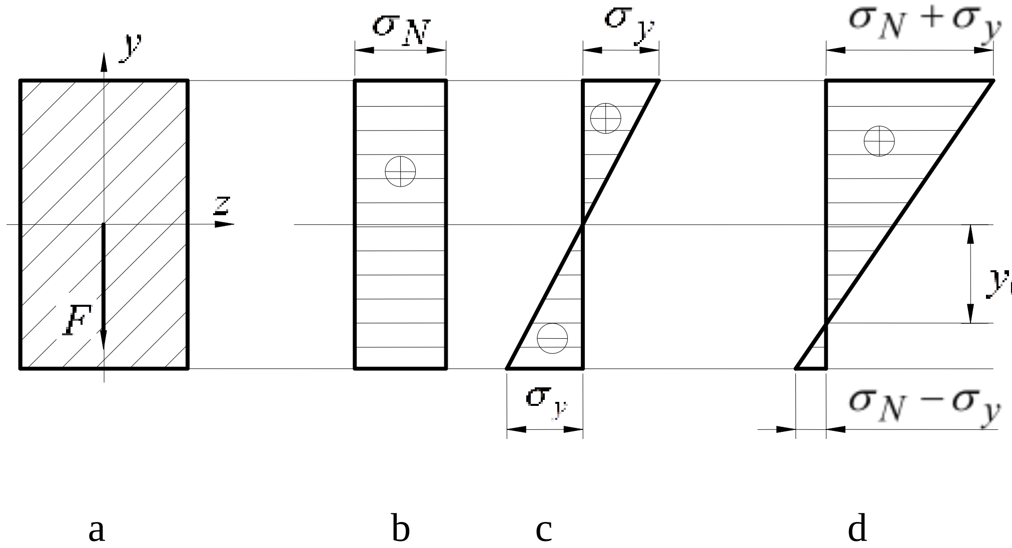

Fig. 26.2

The diagram of normal stresses from the action of normal force N is shown in Fig. 25.2, b, from the bending moment M is shown in Fig. 25.2, c, and the total diagram is shown in Fig. 25.2, d.

From the summary diagram it can be seen that with the simultaneous bending and tension the neutral axis of the cross-section is shifted relative to it's centre.

Let us define a new position of the neutral axis of the cross section relative z_0 to the centroid of the section. Since at points, which are on the neutral axis, the normal stresses are equal to zero, then, based on equation (25.3), we obtain:

$$\frac{N}{A} + \frac{M_x \cdot y}{I_z} = 0,$$

where

$$y_0 = -\frac{N}{A} \cdot \frac{I_z}{M_x}. \qquad (25.4)$$

The total diagram of normal stresses (Fig. 25.2, d) shows that at a given load of the beam, the upper fibbers will have the maximum stresses at y_{max}. In this case, the maximum value of the bending moment will be in the fixing position. The strength condition of the beam, which is based on the magnitude of normal stresses in the dangerous cross-section, in the place of fixing, and will take the form:

$$\sigma_{max} = \pm \frac{N}{A} \pm \frac{M_x}{W_z} \leq [\sigma]. \qquad (25.5)$$

For materials which work equally on tension and on compression and in elements having cross-sectional shapes, in which the angular points are equally distant from the principal axes of inertia (rectangle, double T-beam, square, etc.), the strength verification of beam is carried out by the formula (25.5). For bars which are made of materials that work differently on tension and compression, the strength verification must be carried out separately for tension and compression.

Initially the calculation a transversal section of beam under the action of bending and tension (compression) stresses is carried out without taking into account the effect of normal force. After determining the required cross-sectional dimensions, the strength of the beam is checked taking into account the acting of normal force. If necessary, its size is adjusted by the way of successive approximation. The value of the maximum stress in absolute value should not differ from the permissible stresses by ±5%.

On beginning

25.3 Example of calculation of beam on bend and tension.

The given cantilever beam is shown in fig. 25.3. The beam is loaded by $q = 10$ N/m, $M = 20$ Nm, $\alpha = 60^0$.

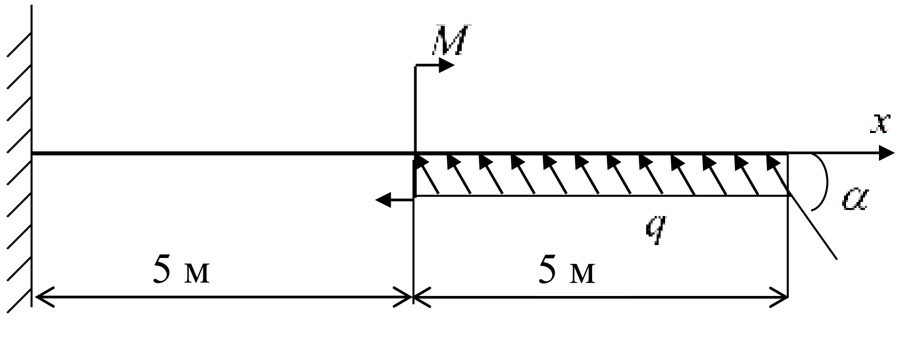

Fig. 26.3

Calculate the double T-beam cross-section of the beam, if $[\sigma]=160$ MPa.

1. Let us build diagram of normal force N_x and diagram of bending moment M_x (рис. 25.4):

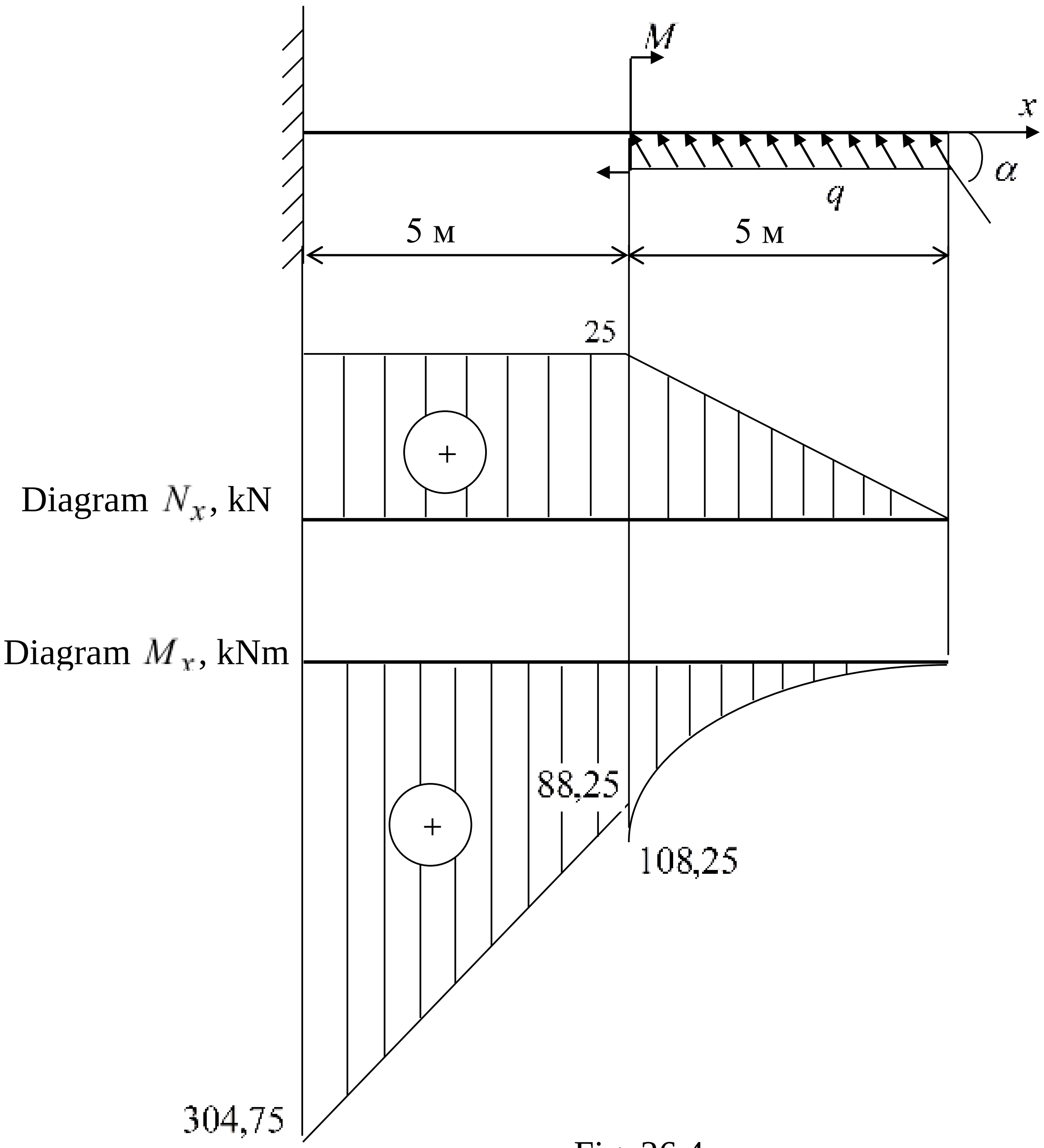

Fig. 26.4

I portion. $0 \le x_1 \le 5$ m.

$N_{x_1} = qx_1 \cos 60^0$;

$x_1 = 0$, $N_{x_1} = 0$; $\qquad x_1 = 5$ m, $N_{x_1} = 25$ kN.

$$M_{x_1} = \frac{qx_1^2}{2} \sin 60^0;$$

$x_1 = 0$, $M_{x_1} = 0$; $\quad x_1 = 5$ m, $M_{x_1} = 108{,}25$ kNm.

II portion $5 \text{ m} \le x_2 \le 10$ m.

$N_{x_2} = q \cdot 5 \cdot \cos 60^0 = 10 \cdot 5 \cdot 0{,}5 = 25$ kN.

$M_{x_2} = -M + q \cdot 5(x_2 - 2{,}5)\sin 60^0;$

$x_2 = 5$ m, $M_{x_2} = 88{,}25$ kNm; $\quad x_2 = 10$ м, $M_{x_2} = 304{,}75$ кН·м.

2. From the diagram it can be seen that the dangerous cross-section will be a place which corresponds to the rigid fixed of the beam, where is $M_{\max} = 304{,}75$ kNm, $N = 25$ kN.

Then, according to formula (25.5), the condition of strength is written as:

$$\sigma_{\max} = \frac{304{,}75 \cdot 10^{-3}}{W_z} + \frac{25 \cdot 10^{-3}}{A} \text{ MPa } \le 160 \text{ MPa}.$$

The condition of strength contains two unknowns W_z and A. Since, as a rule, the bending stress is greater than that one of tension, then at the selection of the cross-section we can first neglect tension. Let us calculate dimensions of the cross-section from the bending. Thus, we obtain:

$$W_z \ge \frac{304{,}75 \cdot 10^{-3}}{160} = 1905 \cdot 10^{-6} \text{ м}^3 = 1905 \text{ см}^3.$$

From the table sorts of steel we choose double T-beam № 55. For it we have: $W_z = 2035$ sm^3, according area $A = 118$ sm^2.

We check the strength of the beam by the formula (25.5):

$$\sigma_{\max} = \frac{304{,}75 \cdot 10^3}{1905 \cdot 10^{-6}} + \frac{25 \cdot 10^3}{118 \cdot 10^{-4}} = (149{,}8 + 2{,}1) \cdot 10^6 \text{ Па}$$

$$= 151{,}9 \text{ МПа}.$$

The beam is not loaded with:

$$\frac{160 - 151{,}9}{160} 100\% = 5{,}1\%.$$

On beginning