Lecture 4
THE METHOD OF CALCULATION THE BAR ON RIGIDITY

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Plan of lecture

- **1.** Introduction
- 2. Consideration of some typical examples
- 3. Thin-walled pressure vessels

Introduction

Normal stresses is described by formula:

$$\sigma = \frac{F}{A}$$

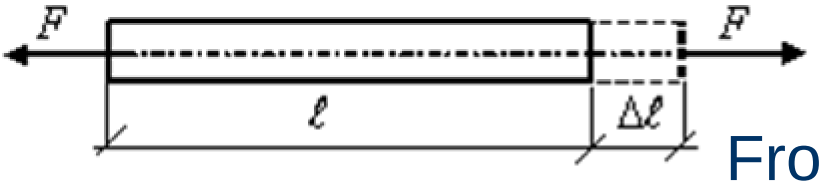


Fig. 1

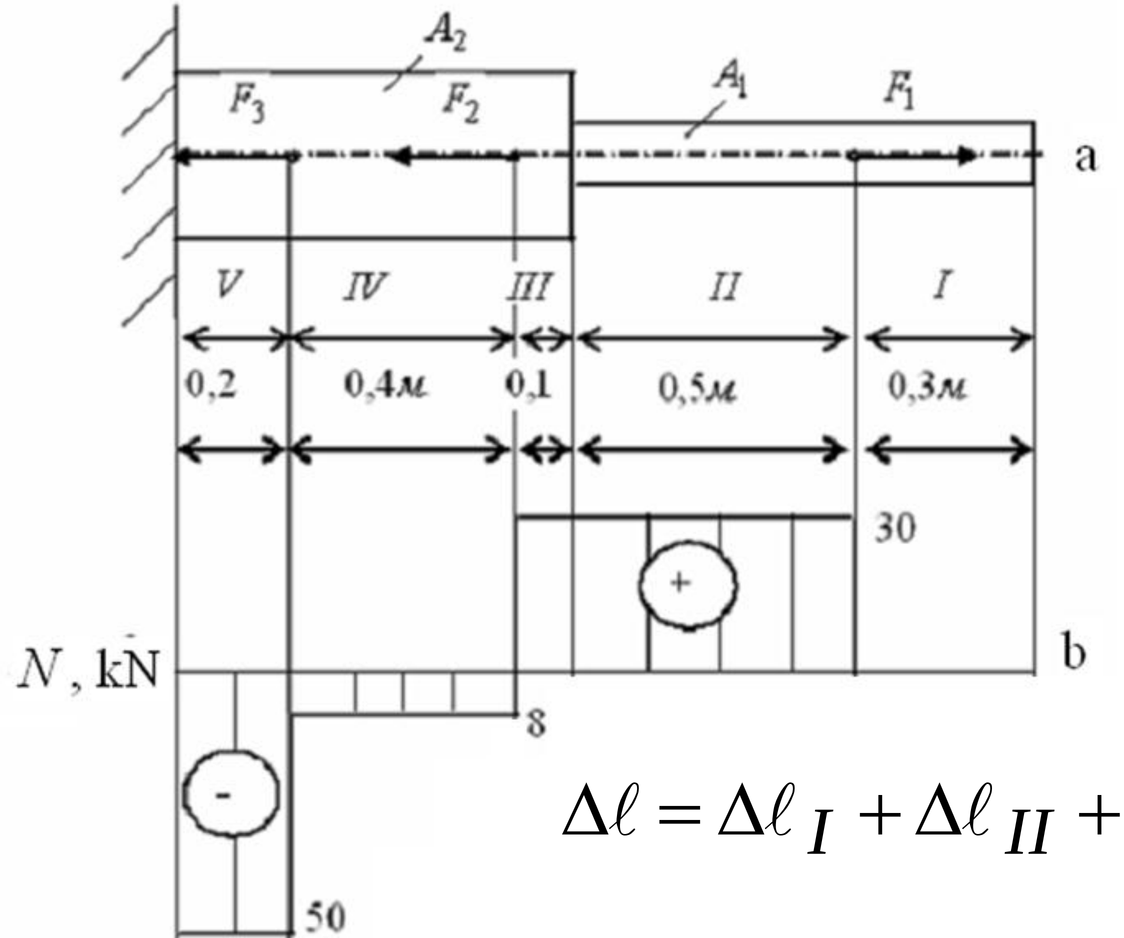
From Hook's Law we could write

$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta\ell/\ell} = \frac{F\ell}{\Delta\ell A}$$

or

$$\Delta\ell = \frac{F\ell}{EA} \qquad \textbf{(1)}$$

Introduction



Let us determine moving of free end of bar, which was considered last lecture

$$\Delta\ell = \Delta\ell_I + \Delta\ell_{II} + \Delta\ell_{III} + \Delta\ell_{IV} + \Delta\ell_V$$

Introduction

Calculate the value of elongation on each of bar portion:

$$\Delta\ell_I = \frac{N_I \cdot \ell_I}{E \cdot A_1} = 0 \qquad \Delta\ell_{II} = \frac{N_{II} \cdot \ell_{II}}{E \cdot A_1} = \frac{30 \cdot 10^3 \cdot 0{,}5 \cdot 10^3}{2 \cdot 10^5 \cdot 1{,}9 \cdot 10^2} = 0{,}394 \text{ mm}$$

$$\Delta\ell_{III} = \frac{N_{III} \cdot \ell_{III}}{E \cdot A_2} = \frac{30 \cdot 10^3 \cdot 0{,}1 \cdot 10^3}{2 \cdot 10^5 \cdot 3{,}1 \cdot 10^2} = 0{,}0484 \text{ mm}$$

$$\Delta\ell_{IV} = \frac{N_{IV} \cdot \ell_{IV}}{E \cdot A_2} = -\frac{8 \cdot 10^3 \cdot 0{,}4 \cdot 10^3}{2 \cdot 10^5 \cdot 3{,}1 \cdot 10^2} = -0{,}0516 \text{ mm}$$

$$\Delta\ell_V = \frac{N_V \cdot \ell_V}{E \cdot A_2} = -\frac{50 \cdot 10^3 \cdot 0{,}2 \cdot 10^3}{2 \cdot 10^5 \cdot 3{,}1 \cdot 10^2} = -0{,}161 \text{ mm}$$

Introduction

Finally we get:

$$\Delta\ell = 0{,}384 + 0{,}0484 - 0{,}0516 - 0{,}161 \cong 0{,}23 \ \textbf{mm}$$

A solid truncated conical bar of circular cross section

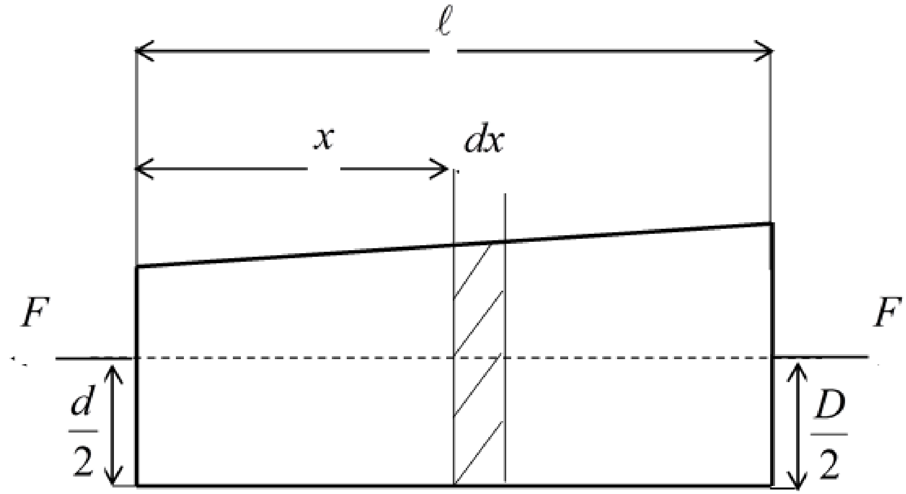


Fig. 2

Introduction

The radius of small element is readily found by similar triangles:

$$r = \frac{d}{2} + \frac{x}{\ell}\left(\frac{D-d}{2}\right)$$

the element, this expression becomes:

$$d\Delta\ell = \frac{F \cdot dx}{\pi\left(\frac{d}{2} + \frac{x}{\ell}\left(\frac{D-d}{2}\right)\right)^2 E}$$

Introduction

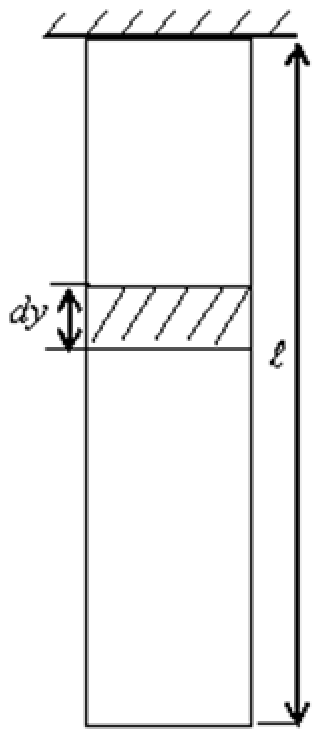


Fig. 3

A bar of constant cross section

$$\Delta\ell = \int_0^L d\Delta\ell = \int_0^L \frac{F \cdot dx}{\pi\left(\frac{d}{2} + \frac{x}{\ell}\left(\frac{D-d}{2}\right)\right)^2 E} = \frac{4FL}{\pi D d E} \qquad \textbf{(2)}$$

The elongation of the element of thickness by shown is**:**

$$d\Delta\ell = \frac{A \cdot y \cdot \gamma}{A \cdot E} dy$$

The total elongation of the bar is:

$$\Delta\ell = \int_0^\ell \frac{A \cdot y \cdot \gamma}{A \cdot E} dy = \frac{A \cdot \gamma}{A \cdot E} \frac{\ell^2}{2} = \frac{A\ell}{2AE}\ell = \frac{W\ell}{2AE} \qquad \textbf{(3)}$$

Consideration of some typical examples

Example 1 A= 500 mm^2 E= 200 GPa

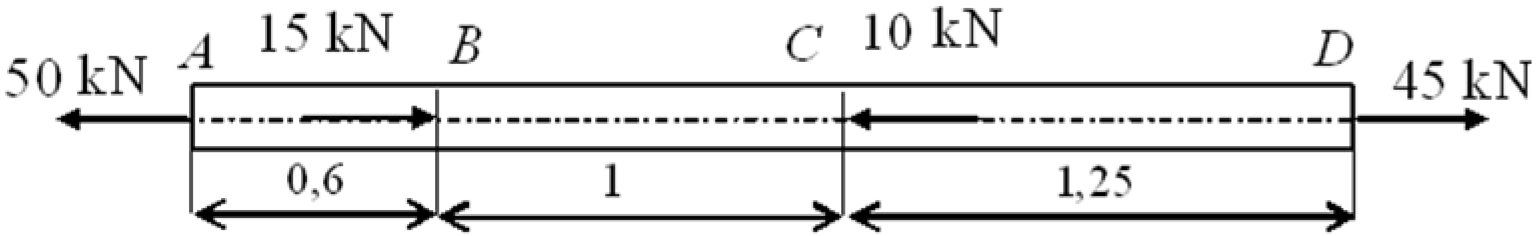


Fig. 4

The elongation of portion AB is:

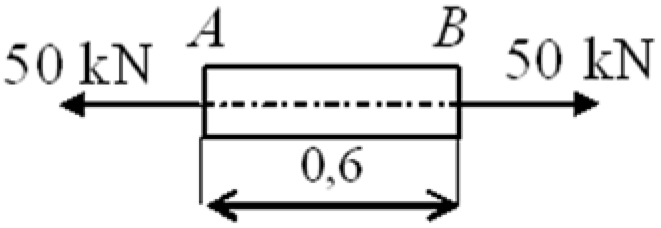


Fig. 5

$$\Delta\ell_1 = \frac{50000 \cdot 0{,}6}{500 \cdot 10^{-6} \cdot 200 \cdot 10^9} = 0{,}0003 \text{ m}$$

Consideration of some typical examples

The elongation of the segment between B and C is :

$$\Delta\ell_2 = \frac{35000 \cdot 1}{500 \cdot 10^{-6} \cdot 200 \cdot 10^{9}} = 0{,}00035 \text{ m}$$

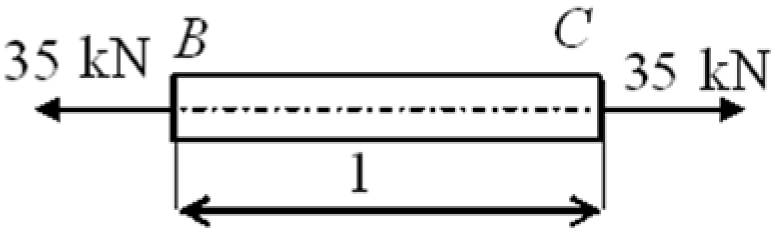


Fig. 6

Consideration of some typical examples

The elongation of CD is:

$$\Delta\ell_2 = \frac{45000 \cdot 1{,}25}{500 \cdot 10^{-6} \cdot 200 \cdot 10^9} = 0{,}00056 \text{ m}$$

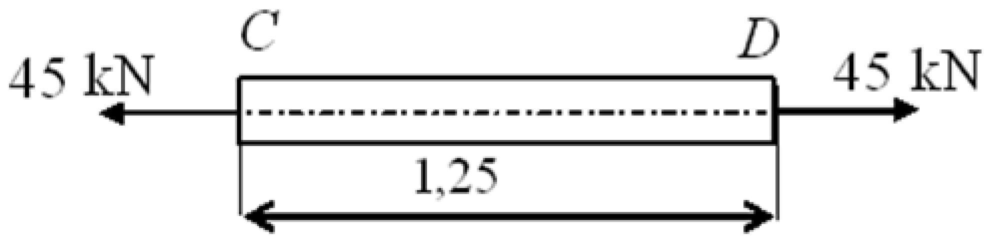


Fig. 7

The total elongation is:

$$\Delta\ell = \Delta\ell_1 + \Delta\ell_2 + \Delta\ell_3 = 0{,}00121 \text{ m}$$

Consideration of some typical examples

Example 2

In 1989, Jason, a research-type submersible with remote TV monitoring capabilities and weighing 35,200 N was lowered to a depth of 646 m in an effort to send back to the attending surface vessel photographs of a sunken Roman ship offshore from Italy.

The submersible was lowered at the end of a hollow steel cable having an area of m^2 and GPa. The central core of the cable contained the fiber-optic system for transmittal of photographic images to the surface ship.

Consideration of some typical examples

The total cable extension is the sum of the extensions due to (1) the weight of Jason:

$$\Delta\ell_1 = \frac{F\ell}{EA} = \frac{35{,}2 \cdot 646}{452 \cdot 10^{-6} \cdot 200 \cdot 10^{9}} = 0{,}252 \text{ m}$$

From (3) we have for the weight of the steel cable:

$$\Delta\ell_2 = \frac{W\ell}{2AE}$$ where W is the weight of the cable

Weight may be found as the volume of the cable:

$$452 \cdot 10^{-6} \cdot 646 = 0{,}292 \text{ m}^3$$

Consideration of some typical examples

Thus, the cable weight is:

$$W = 0{,}292 \cdot 77 = 22{,}484 \quad \text{N}$$

The elongation due to the weight of the cable is:

$$\Delta\ell_2 = \frac{22{,}484 \cdot 646}{2\left(452 \cdot 10^{-6} \cdot 200 \cdot 10^{9}\right)} = 0{,}080 \quad \text{m}$$

The total elongation is:

$$\Delta\ell = \Delta\ell_1 + \Delta\ell_2 = 0{,}252 + 0{,}080 = 0{,}332 \quad \text{m}$$