Lecture 13.
STATE OF STRESS AT A POINT

Assos. Prof. A. Kutsenko

Plan of lecture

- **1. The stress at a point**
- **2. About stress tensor**
- **3. Theorem of the shearing stresses equivalence**

The stress at a point

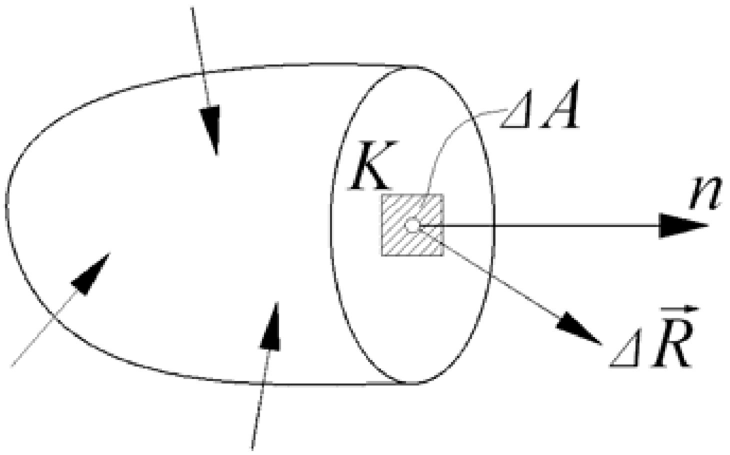

Fig.1

A small area Δ*A* around point K in boundary plane of the left body portion is considered.

Furthermore, it is supposed that the main vector Δ*R* correctly describes the state of internal forces on the small area Δ*A* around the point.

The stress at a point

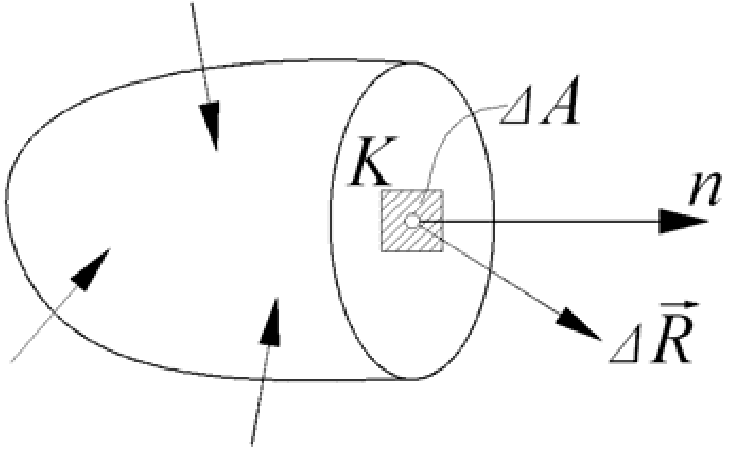

Fig.1

The *average stress* on the area can be described by the expression

$$p = \frac{\Delta R}{\Delta A} \qquad (1)$$

Furthermore, it is supposed that the main vector Δ*R* correctly describes the state of internal forces on the small area Δ*A* around the point.

The stress at a point

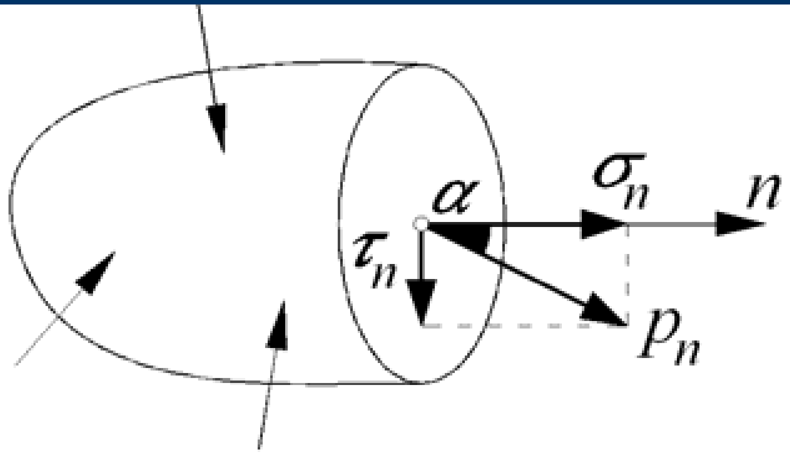

Fig.2

Thus, the limit transition ***ΔA → 0*** can be used. Then, ***stress on the area of normal n around point K is***

$$p_n = \lim_{\Delta A \to 0} \frac{\Delta R}{\Delta A} = \frac{dR}{dA} \qquad (2)$$

Vector of stress depends on the surface forces, body forces, on the position of the point considered, and on the orientation of the area around the point.

The stress at a point

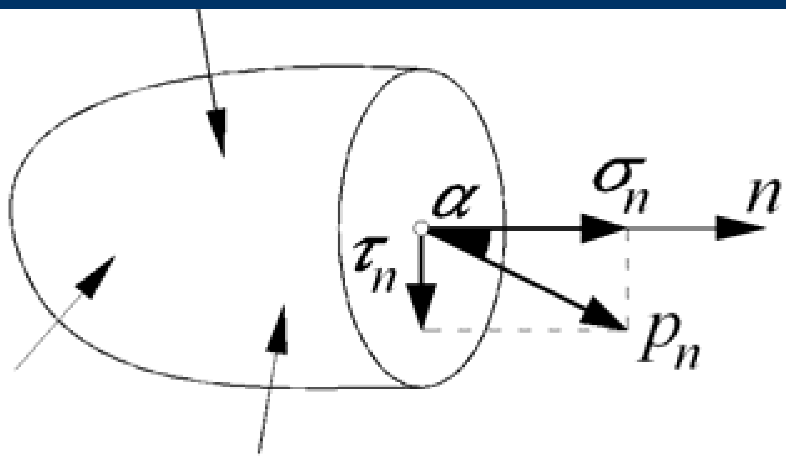

Fig.2

$$\sigma_n = p_n \cos\alpha$$

$$\tau_n = p_n \sin\alpha \qquad (3)$$

Generally, the stress vector is inclined at an angle with respect to the plane of the cross-section. Let α to be the angle between stress *pn* and the cross-sectional normal *n*. Then,

The stress at a point

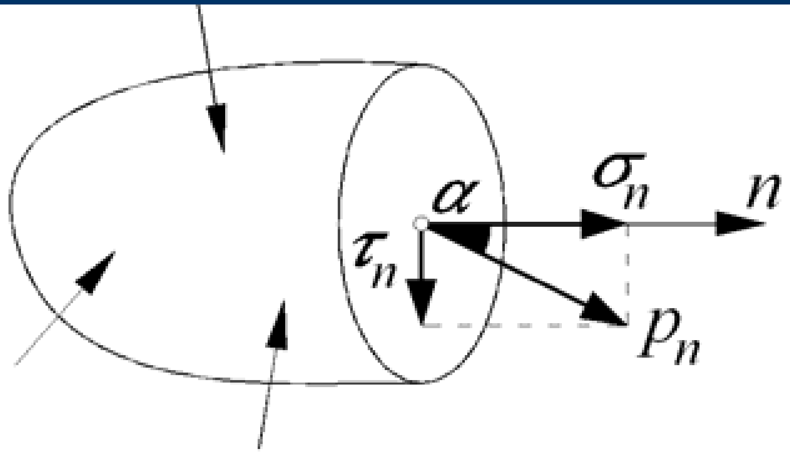

Fig.2

$$\sigma_n = p_n \cos\alpha$$

$$\tau_n = p_n \sin\alpha \qquad (3)$$

The normal stresses arise when the particles of the body strive either to remove or to approach each other. Shearing stresses are related to the mutual displacements of the particles in the crosssectional plane

The stress at a point

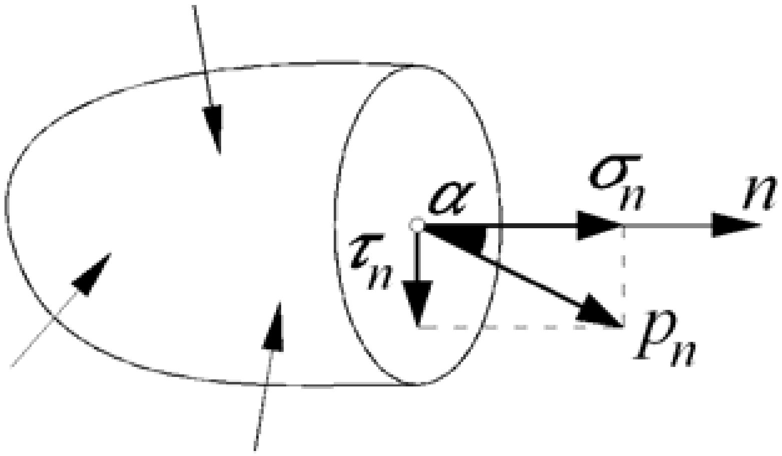

Fig.2

It is evident that

$$p_n = \sqrt{\sigma_n^2 + \tau_n^2} \qquad \textbf{(4)}$$

About stress tensor

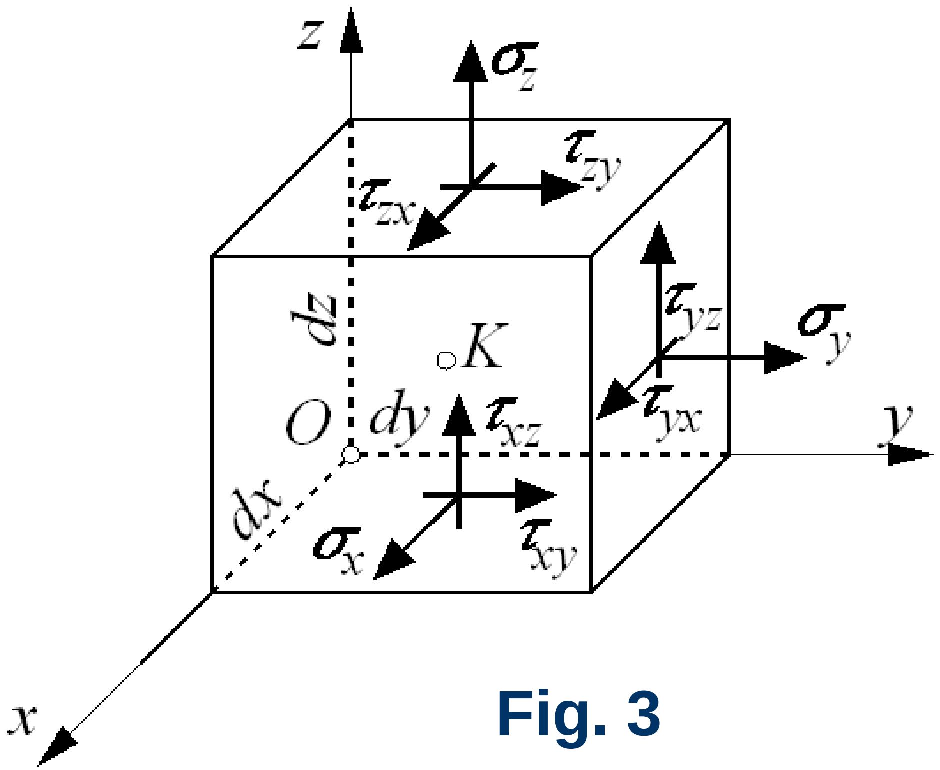

Fig. 3

A body loaded by a set of external forces is given. An infinitesimal parallelepiped of dimensions *dx, dy, dz* in the vicinity of arbitrary chosen point of the body is separated. The normal and shearing stresses about the point investigated will act on the walls of the parallelepiped.

About stress tensor

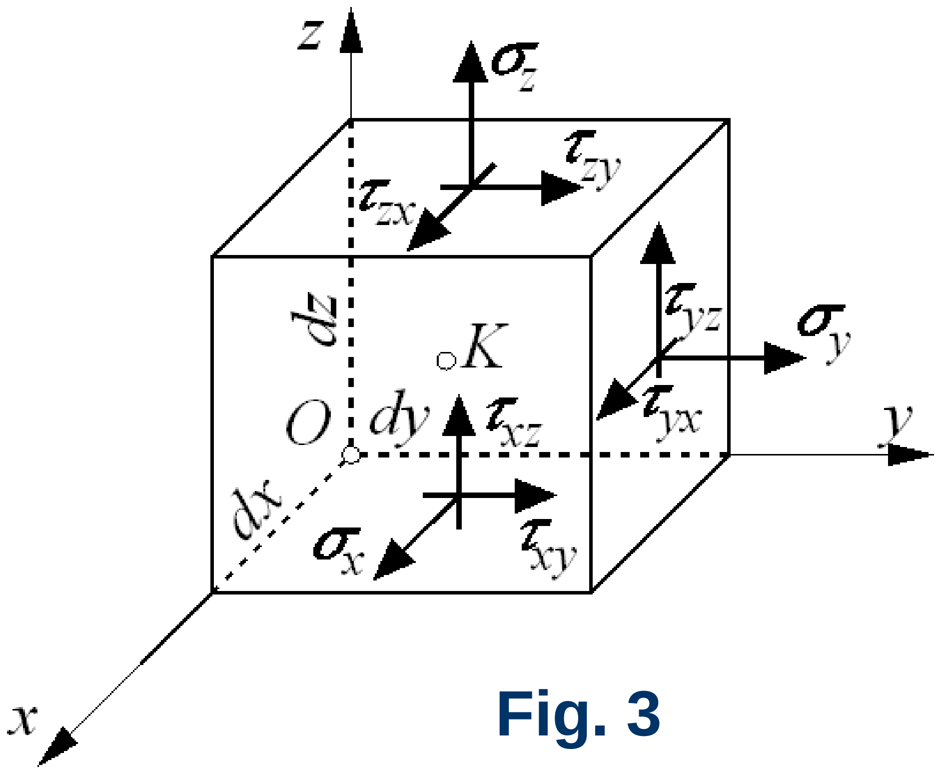

Fig. 3

Normal stresses **are written with *one index*. It corresponds to the letter of the coordinate axis parallel to the normal stress considered.**

About stress tensor

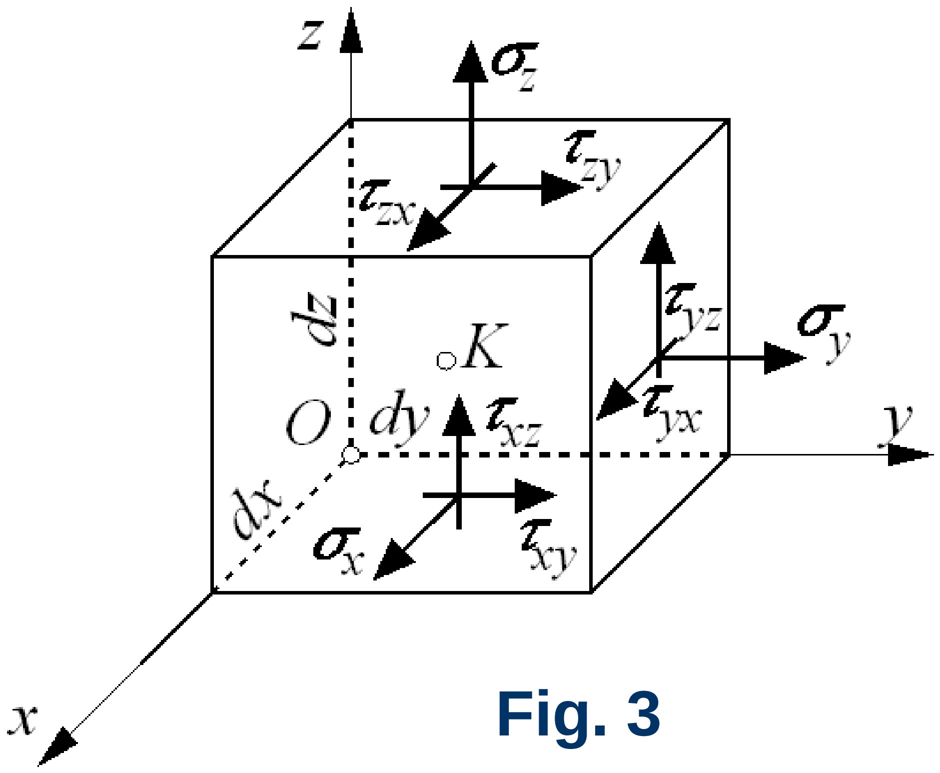

Fig. 3

Shearing stresses **have two indices.** ***The first one*** **corresponds to the index of the normal stress of this wall while** ***the second index*** **is the letter of the coordinate axis parallel to the shearing stress considered.**

About stress tensor

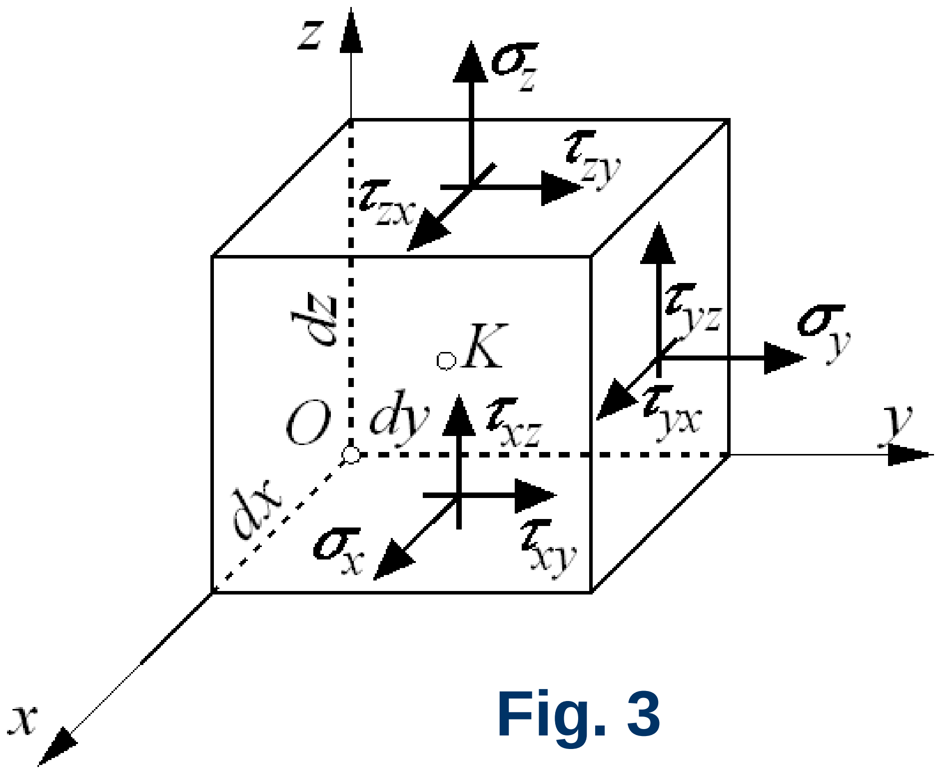

Fig. 3

The behavior of the body acted upon by external forces does not depend on the coordinate system. Therefore, the state of stress can be described by *tensor*

About stress tensor

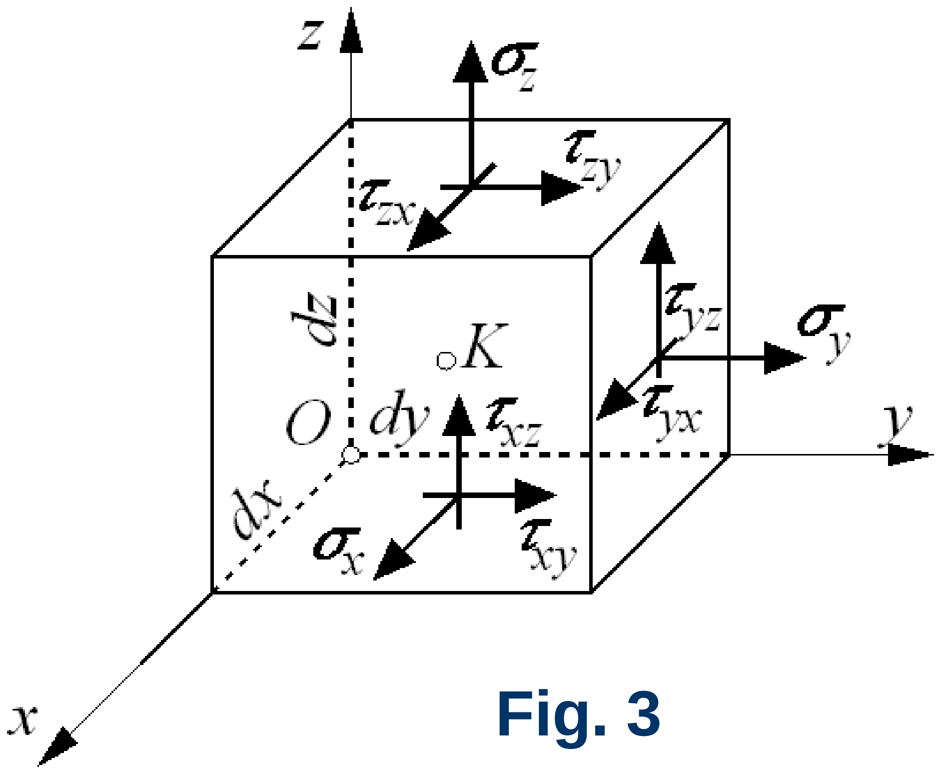

Fig. 3

stress tensor

$$\sigma_{ij} = \begin{vmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{vmatrix} \quad \textbf{(5)}$$

Theorem of the shearing stresses equivalence

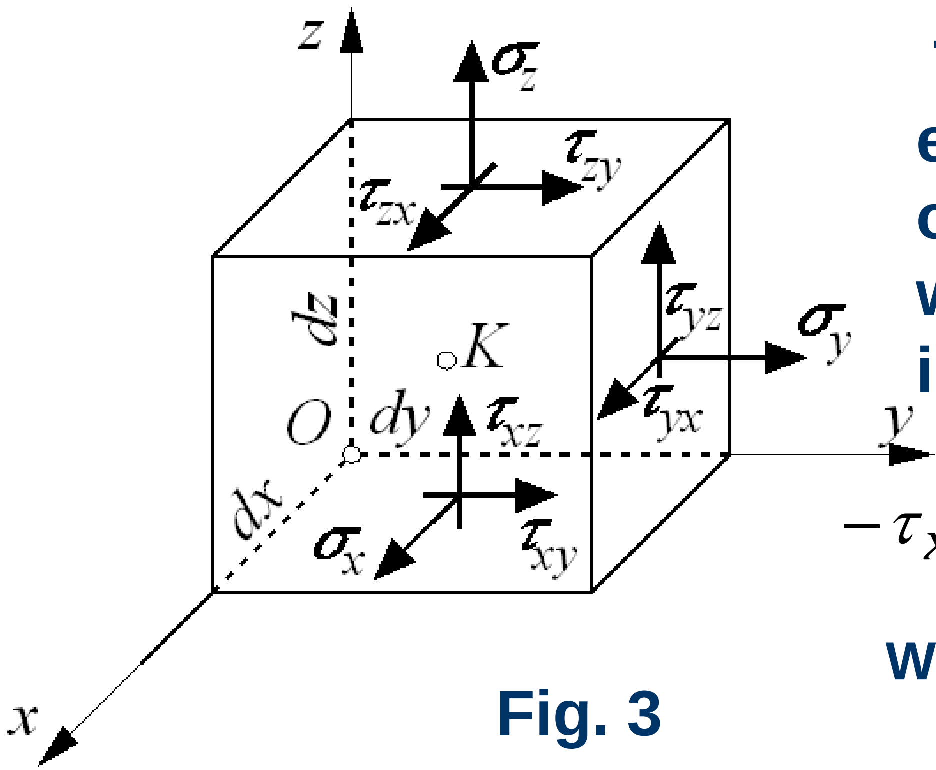

Fig. 3

The moment equation of equilibrium about *z*-axis of the forces loading the walls of the arallelepiped is

$$-\tau_{xy} dydzdx - \tau_{yx} dxdzdy = 0$$

We will get $\tau_{xy} = \tau_{yx}$

In the same way $\tau_{yz} = \tau_{zy}$ **and** $\tau_{zx} = \tau_{xz}$

Theorem of the shearing stresses equivalence

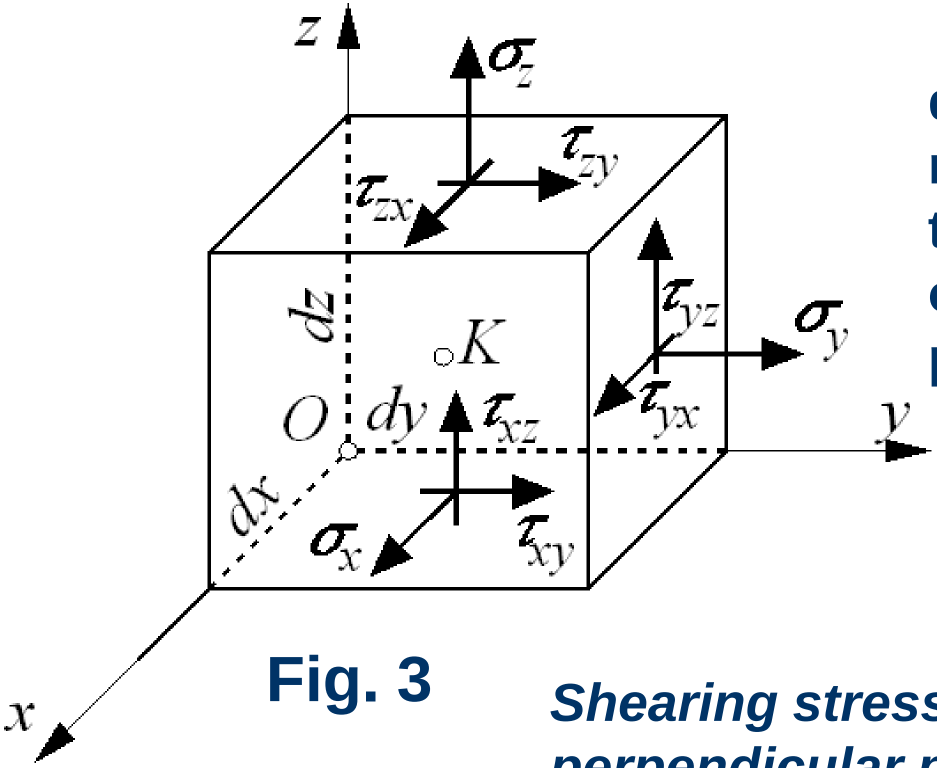

Fig. 3

The theorem gives the dependence between the magnitudes and directions of the shearing stresses acting on two mutually perpendicular planes around a point.

Shearing stresses on two mutually perpendicular planes are equals. They are either “meeting” or “running” to each other.

Theorem of the shearing stresses equivalence

stress tensor

$$\sigma_{ij} = \begin{vmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{vmatrix}$$

Applying the theorem of the shearing stresses equivalence, it can be concluded that only six stresses are independent of each other. These six parameters define the state of stress at point *K*.

Thank you!

Good bye!