Lecture 15.
CALCULATION OF SIMPLY SUPPORTED BEAMS

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Plan of lecture

- 1. The determination of the reactions of beam supports
- 2. The building of diagrams of the shearing forces and the bending moments for given beam
- 3. The selection of cross-section for a given beam

The determination of the reactions of beam supports

Given

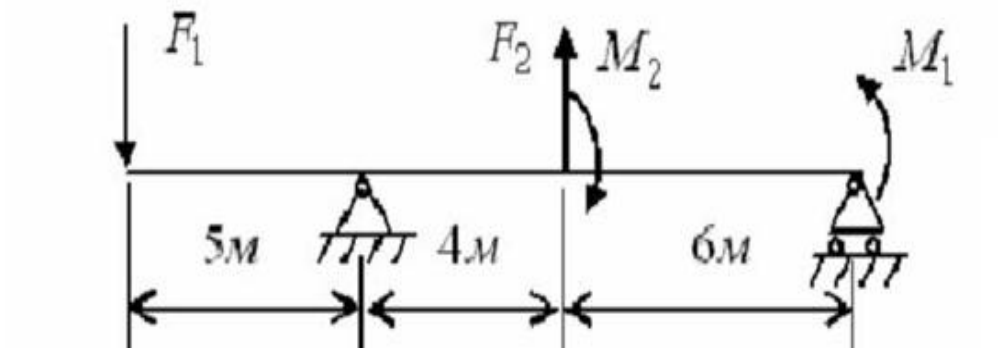

Fig.1

$$[\sigma] = 160 \text{ MPa}$$

$$F_1 = 18 \text{ kN}$$

$$F_2 = 30 \text{ kN}$$

$$M_1 = 20 \text{ kNm}$$

$$M_2 = 10 \text{ kNm}$$

The determination of the reactions of beam supports

From statics we have: $\sum_{i=1}^{n} F_{ix} = 0 \quad R_{Bx} \equiv 0$

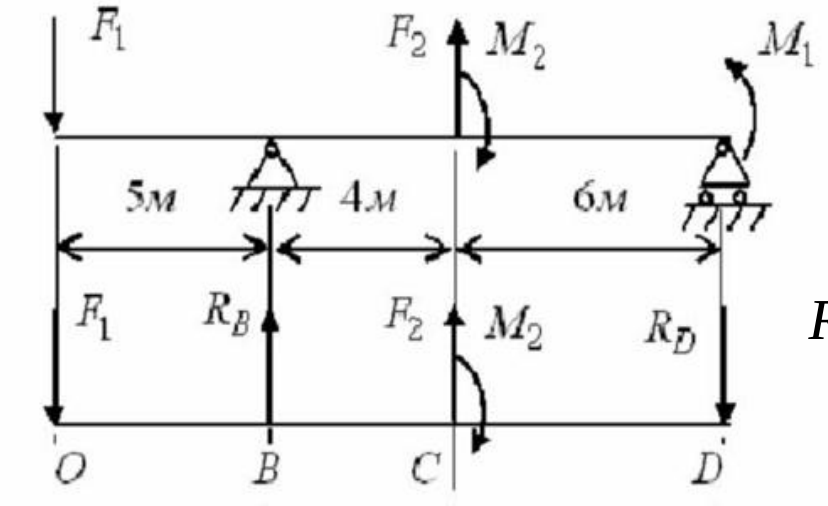

Fig. 2

$$\sum M_D = 0$$

$$R_{B_y} = \frac{M_1 - F_2 \cdot CD - M_2 + F_1 \cdot OD}{BD} =$$

$$= \frac{20 - 30 \cdot 6 - 10 + 18 \cdot 15}{10} = 10$$

kN

The determination of the reactions of beam supports

From statics we have: $\sum M_B = 0$

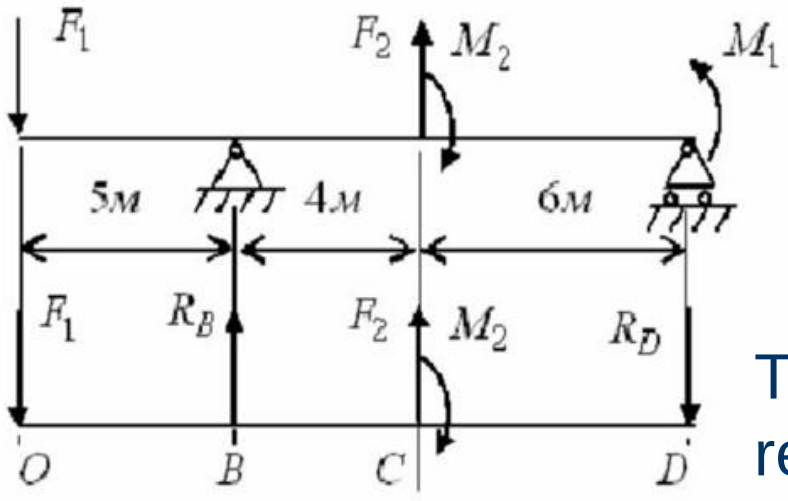

Fig. 2

$$R_D = \frac{-M_1 - F_2 \cdot BC + M_2 - F_1 \cdot OB}{BD} =$$

$$= \frac{-20 - 30 \cdot 4 + 10 - 18 \cdot 5}{10} = -22 \ \textbf{kN}$$

The verification of obtained reactions: $\sum_{i=1}^{n} F_{iy} = 0$

$$-F_1 + R_{By} - R_D + F_2 = -18 + 10 - 22 + 30 = -8 + 8 = 0$$

The building of diagrams of the shearing forces and the bending moments

Let us divide a beam into portions by characteristic cross - sections O, B, C, D

Definition the value of shear force

$$Q_{y_O}^{rt} = -F_1 = -18 \text{ kN} \qquad Q_{y_O}^{lt} = -F_1 = -18 \text{ kN}$$

$$Q_{y_B}^{rt} = -F_1 + R_B = -18 + 10 = -8 \text{ kN}$$

$$Q_{y_C}^{lt} = -F_1 + R_B = -18 + 10 = -8 \text{ kN}$$

$$Q_{y_C}^{rt} = -F_1 + R_B + F_2 = -18 + 10 + 30 = 22 \text{ kN}$$

$$Q_{y_D}^{lt} = -F_1 + R_B + F_2 = -18 + 10 + 30 = 22 \text{ kN}$$

The building of diagrams of the shearing forces and the bending moments

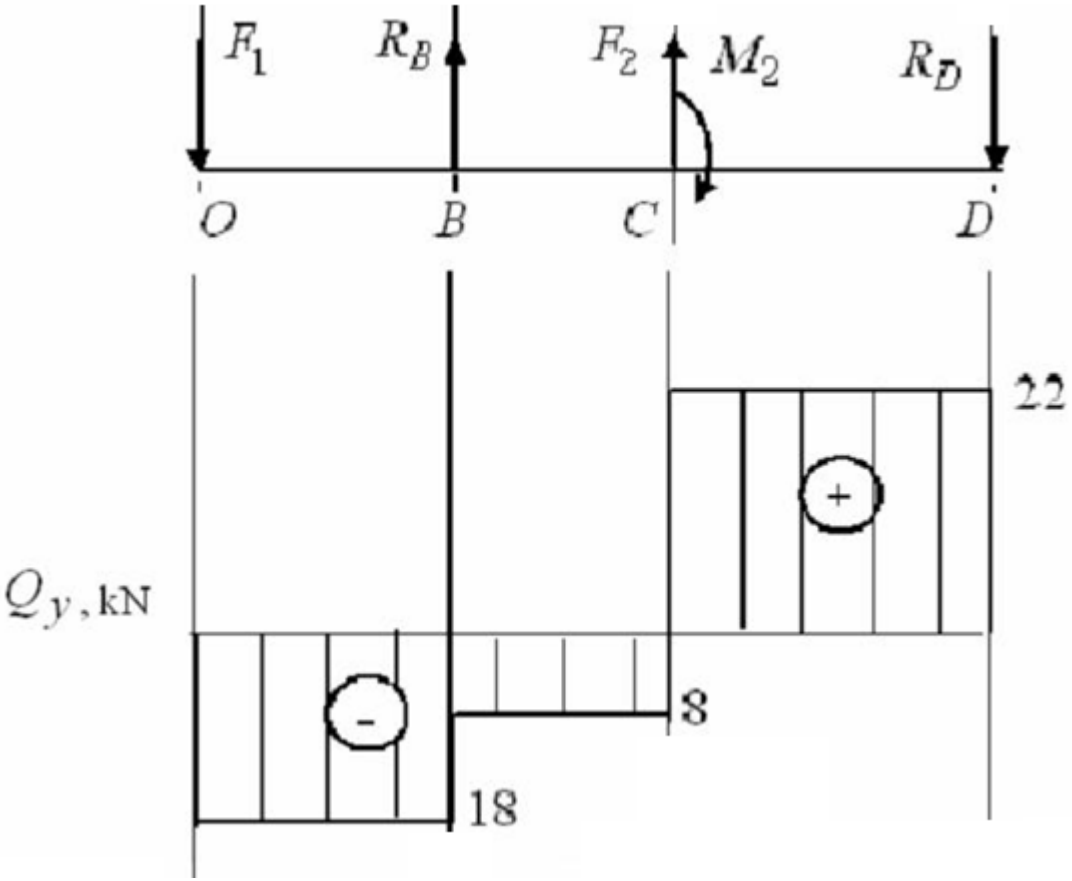

Diagram of shear force Qy

Fig. 3

The building of diagrams of the shearing forces and the bending moments

Definition the value of bending moment $M_0 = 0$

$$M_B = -F_1 \cdot AB = -18 \cdot 5 = -90 \text{ kNm}$$

$$M_B^{lt} = -F_1 \cdot OC + R_B \cdot BC = -18 \cdot 9 + 10 \cdot 4 = -122 \text{ kNm}$$

$$M_B^{lt} = -F_1 \cdot OC + R_B \cdot BC + M_2 =$$

$$= -18 \cdot 9 + 10 \cdot 4 + 10 = -112 \text{ kNm}$$

$$M_D^{lt} = -F_1 \cdot OD + R_B \cdot BD + M_2 + F_2 \cdot CD =$$

$$= -18 \cdot 15 + 10 \cdot 10 + 10 + 306 = 20 \text{ kNm}$$

The building of diagrams of the shearing forces and the bending moments

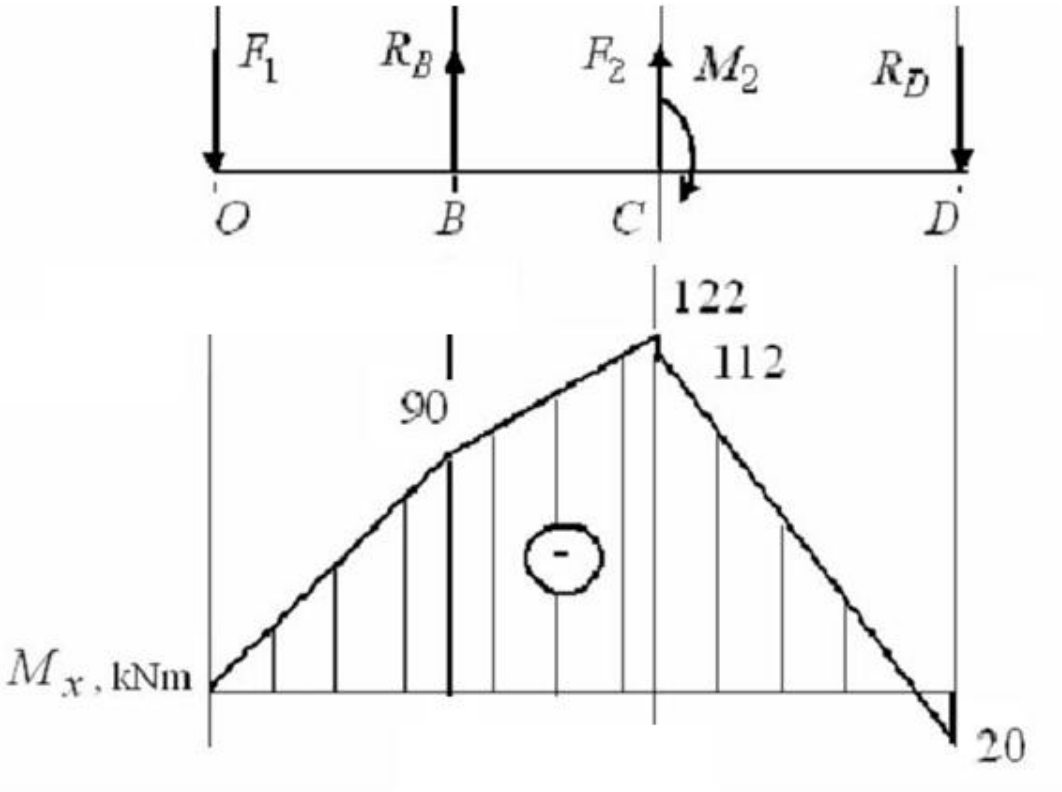

Diagram of bending moment Mx

Fig. 4

The selection of cross-section

The section of rectangle cross - section:

$$W_x = \frac{M_{x\max}}{[\sigma]} = \frac{122 \cdot 10^3}{160 \cdot 10^6} = 0{,}762 \cdot 10^{-3} \text{ m}^3$$

Taking into account, that $h = 1{,}5b$, we find that:

$$b = \sqrt[3]{\frac{6W_x}{2{,}25}} = \sqrt[3]{\frac{6 \cdot 0{,}762 \cdot 10^6}{2{,}25}} = 127 \text{ mm}$$

The selection of cross-section

The section of circle cross - section:

$$W_x = \frac{\pi d^3}{32}$$

we find a diameter of circle:

$$d = \sqrt[3]{\frac{32W_x}{\pi}} = \sqrt[3]{\frac{32 \cdot 0{,}762 \cdot 10^6}{3{,}14}} = 196\,\textbf{mm}$$

Thank you!

Good bye!