Lecture 10.
THE CALCULATION OF THE BAR ON STRENGTH AND RIGIDITY BY TORSION

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Plan of lecture

- 1. The determination of the inside and outside diameter of the shaft from on rigidity by torsion
- 2. Typical example of calculation of shaft by torsion
- 3. The statically indeterminate problem under torsion

The determination of the inside and outside diameter of the shaft

Given shaft has:

L=3 m **T=25 kNm**

G = 3 GPa $\left[\tau\right]=90$ **MPa**

Determine the inside and outside diameter of the shaft

The determination of the inside and outside diameter of the shaft

The angle of twist is equal: $\theta = {T\ell}/{GI_\rho}$

Thus, in the 3-m length we have:

$$\frac{2,5}{57,3} = \frac{25000 \cdot 3}{85 \cdot 10^9 \frac{\pi}{32}\left(D^4 - d^4\right)}$$

or:

$$D^4 - d^4 = 206 \cdot 10^{-6}$$ **m³**

The determination of the inside and outside diameter of the shaft

From Eq. (9.2), we get:

$$90 \cdot 10^6 = \frac{25000 \cdot D/2}{\frac{\pi}{32}\left(D^4 - d^4\right)}$$

or:

$$D^4 - d^4 = 1414 \cdot D \cdot 10^{-6}$$

Comparison of the right-hand sides of these equations indicates that:

$$206 \cdot 10^{-6} = 1414 \cdot D \cdot 10^{-6}$$

The determination of the inside and outside diameter of the shaft

Thus, finally we get that:

$$D = 0{,}145 \quad \textbf{m}$$

or:

$$D = 145 \quad \textbf{mm}$$

Substitution of this value into either of the equations then gives:

$$d = 0{,}125 \quad \textbf{m}$$

or:

$$d = 125 \quad \textbf{mm}$$

Typical example of calculation of shaft by torsion

Given shaft has:

$M_1 = M_2 = M_3 = M_4 = 150$ **Nm**

$[\tau] = 55$ **kPa**

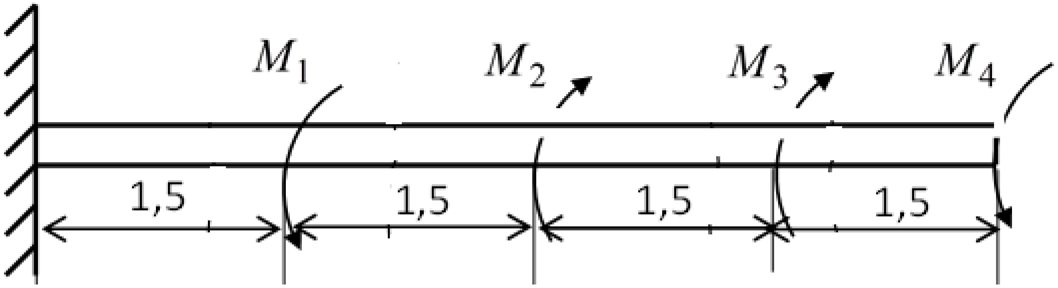

Fig. 1

Typical example of calculation of shaft by torsion

Drawing the diagram of torques T

I portion $0 \le x_1 \le 1{,}5$ **m** $T_1 = M_4 = 150$ **Nm**

II portion $1{,}5 \le x_2 \le 3$ **m** $T_2 = M_4 - M_3 = 0$ **Nm**

III portion $3 \le x_3 \le 4{,}5$ **m**

$$T_3 = M_4 - M_3 - M_2 = -150 \text{ Nm}$$

IV portion $4{,}5 \le x_4 \le 6$ **m**

$$T_4 = M_4 - M_3 - M_2 + M_1 = 0$$

Typical example of calculation of shaft by torsion

Diagram of torques T

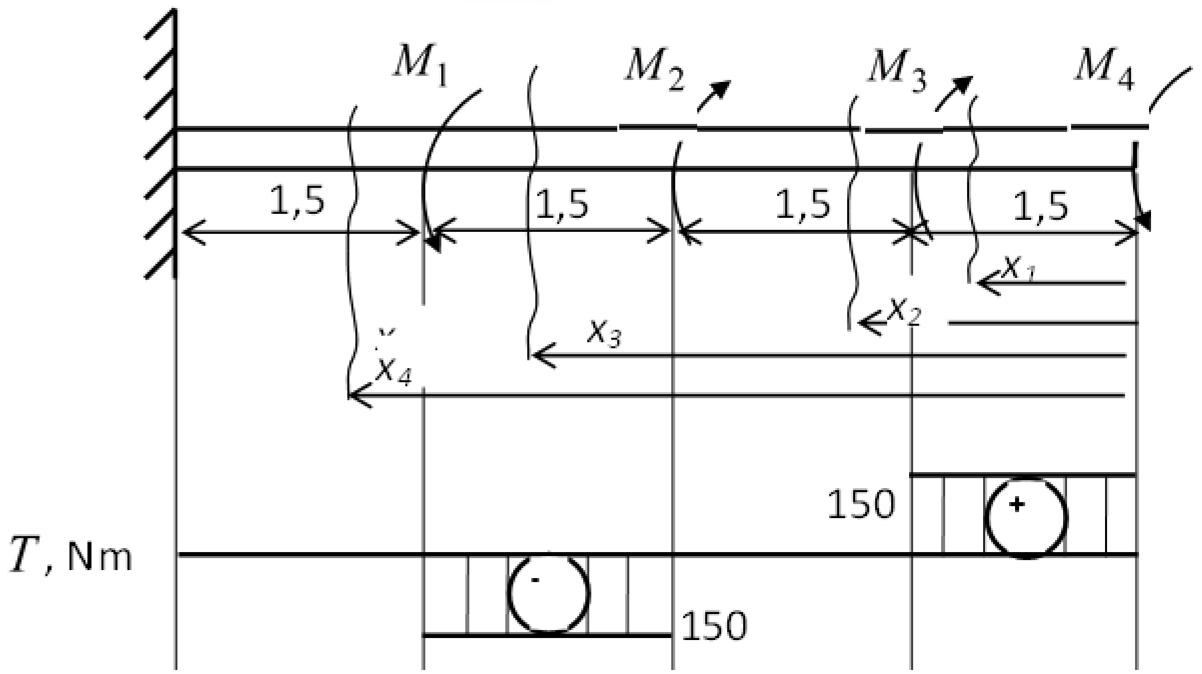

Fig. 2

Typical example of calculation of shaft by torsion

From the condition of durability the diameter of shaft is:

$$d \geq \sqrt[3]{\frac{16 \cdot T_{\max}}{\pi \cdot [\tau]}}$$

in this case we have: $T_{\max} = 150$ **Nm**

so

$$d \geq \sqrt[3]{\frac{16 \cdot 150}{3{,}14 \cdot 55 \cdot 10^3}} \approx 0{,}024 \text{ m}$$

Thus, d = 30 mm

and

$$I_\rho = \frac{\pi d^4}{32} = \frac{3{,}14 \cdot 3^4}{32} \approx 7{,}95 \text{ cm}$$

Typical example of calculation of shaft by torsion

Drawing the diagram of the angle of twist

I portion $0 \le x_1 \le 1{,}5$ **m** $\varphi_1 = 0$

II portion $1{,}5 \le x_2 \le 3$ **m**

$$\varphi_2 = \frac{-150 \cdot 1{,}5}{8{,}1 \cdot 10^{10} \cdot 7{,}95 \cdot 10^{-8}} \approx 3{,}49 \cdot 10^{-2} \text{ rad}$$

III portion $3 \le x_3 \le 4{,}5$ **m**

$$\varphi_3 = \varphi_2 = 3{,}49 \cdot 10^{-2} \text{ rad}$$

Typical example of calculation of shaft by torsion

IV portion $4{,}5 \le x_4 \le 6$ **m** $\varphi_4 = \varphi_3 + \dfrac{150 \cdot 1{,}5}{8{,}1 \cdot 10^{10} \cdot 7{,}95 \cdot 10^{-8}} = 0$

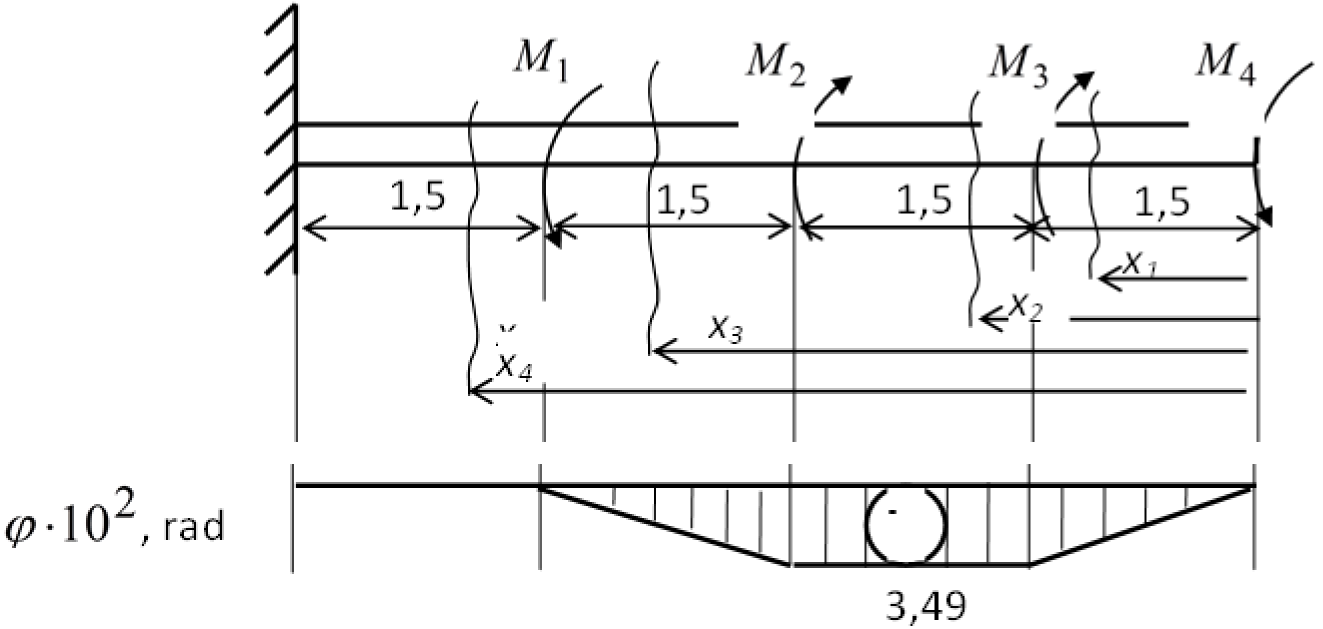

Fig. 3

The statically indeterminate problem under torsion

Let us determine the reactive torques at the fixed ends of the circular shaft loaded by the couples

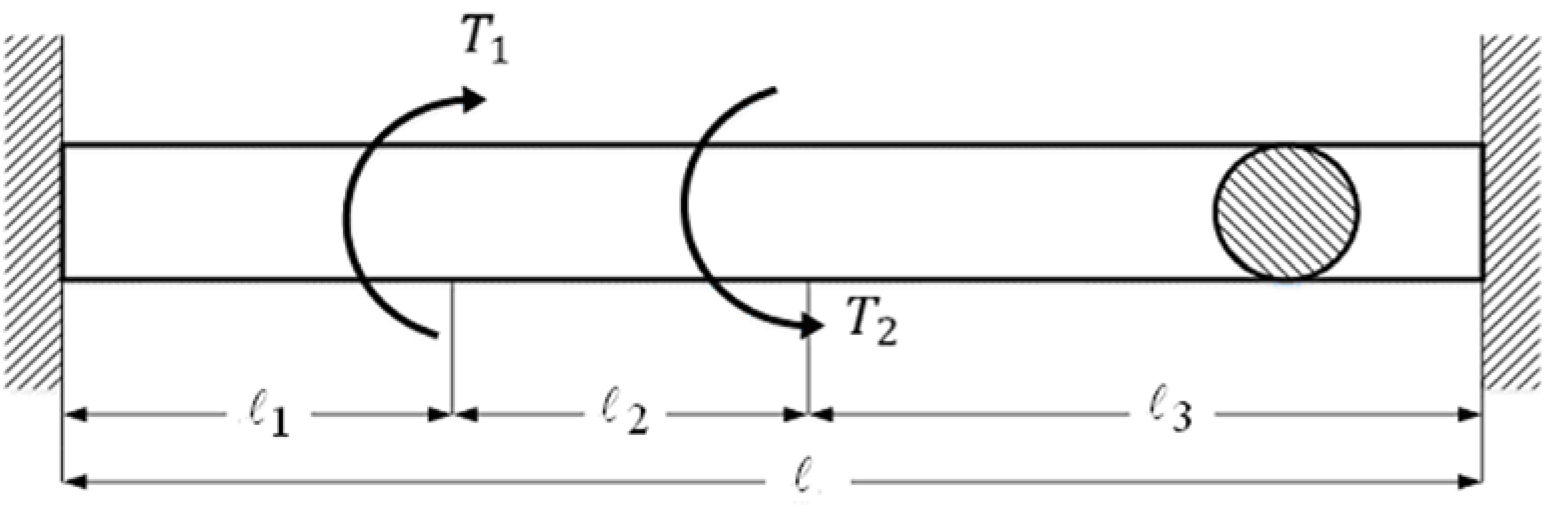

Fig. 4

The statically indeterminate problem under torsion

Let us assume that the reactive torques and are positive in the directions shown in Fig. 5.

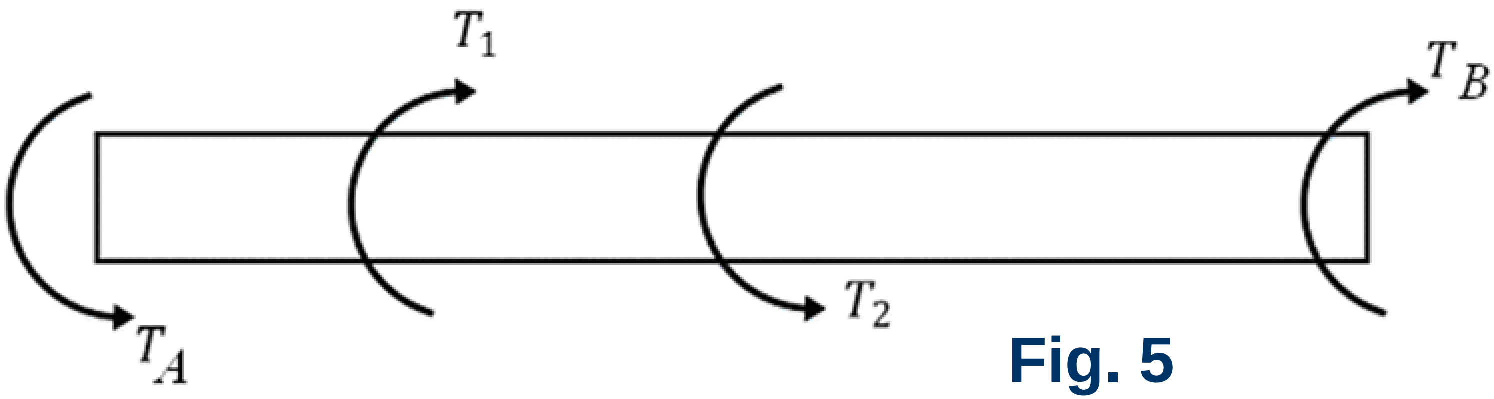

Fig. 5

From static we have:

$$T_A - T_1 + T_2 - T_B = 0$$

The statically indeterminate problem under torsion

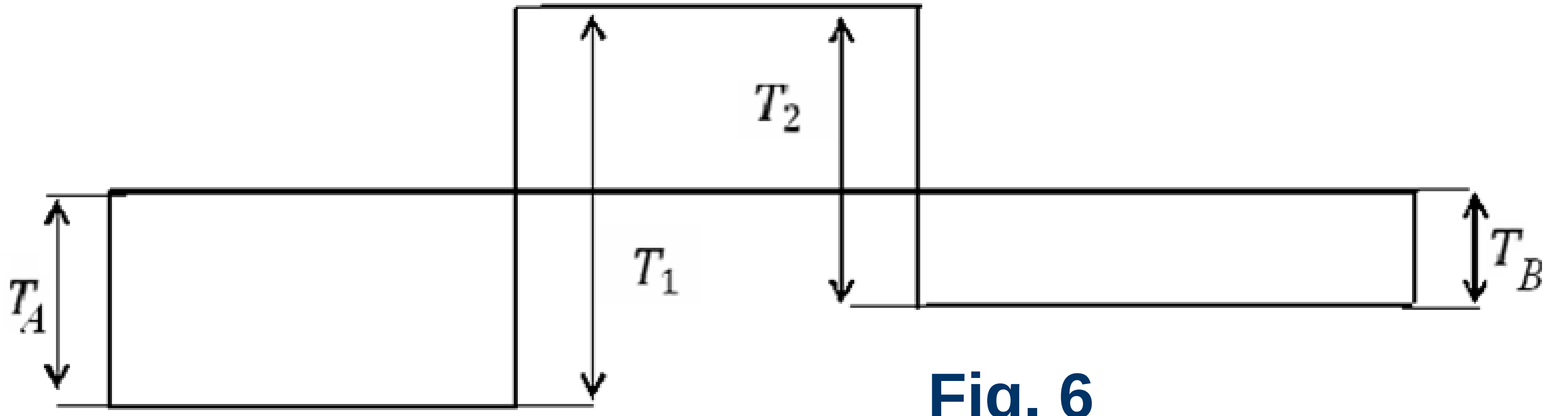

Fig. 6

we immediately have:

$$\theta_1 = \frac{T_A \ell_1}{GI_\rho} \quad \textbf{and} \quad \theta_2 = \frac{T_B \ell_3}{GI_\rho}$$

The statically indeterminate problem under torsion

Hence, since the torque causing this deformation is $T_1 - T_A$, we have

$$\theta_1 + \theta_2 = \frac{(T_1 - T_A)\ell_2}{GI_\rho}$$

we find:

$$T_A = T_1 \frac{\ell_2 + \ell_3}{\ell} \quad \textbf{and} \quad T_B = -T_1 \frac{\ell_1}{\ell} + T_2 \frac{\ell_1 + \ell_2}{\ell}$$

Thank you!

Good bye!