Lecture 22
THE DIAGRAMS OF EXTERNAL EFFORTS OF CURVETTED BEAMS

Plan
1. The building the diagram of normal force.
2. The building the diagram of shear force.
3. The building the diagram of bend moment.

22.1. The building the diagram of normal force.

Now it is logical to consider the rules of building the diagrams of internal forces for the curvetted beams.

In principle, curvetted beams these are the same beams, which longitudinal axis has a curvature. That is why there is a type of curvetted beams, for which there is no necessary for a preliminary determination of support reactions to building diagrams of the internal forces. The curvetted beams which are among to this type, the curvetted beams one end of which is fixed rigidly, and the other is free. It is on the example of such a beam that we begin to consider the problem of building an internal effort for simple curvetted beams.

Consequently, we consider the principle of building of diagram of normal forces, shearing forces and bending moments on the example of curvetted beam, shown in Fig. 22.1. As noted above, it does not require a procedure for determining the supports reactions

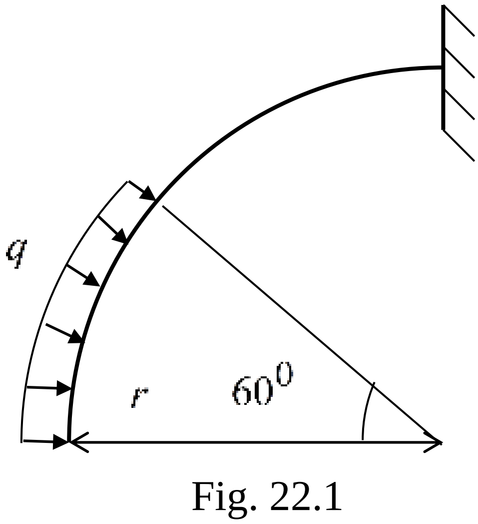

Fig. 22.1

First we will draw a diagram of normal forces $N\varphi$ (Fig. 22.4). To do this, consider two portions of the beam. In each portion, the external forces applied to the beam will be projected in the direction of the tangent at the point of the imaginary transversal section of the beam.

For definitely, $q = 10$ kN/m, $r = 2$ m are given.

I portion. $0 \le \varphi_1 \le 60^0$ (fig. 22.2):

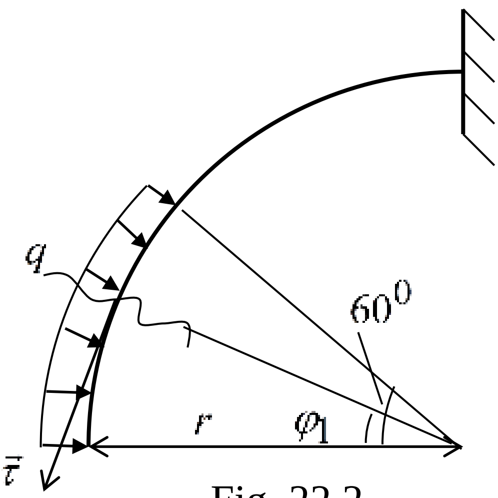

Fig. 22.2

$$N_{\varphi_1} = -2q \cdot r \cdot \sin^2 \frac{\varphi_1}{2} = -q \cdot r(1 - \cos\varphi_1) = -20(1 - \cos\varphi_1);$$

$$N_{\varphi_1}(0) = 0;\ N_{\varphi_1}\left(60^0\right) = -20 \cdot 0{,}5 = -10 \text{ kN}.$$

II portion. $60^0 \le \varphi_2 \le 90^0$ (fig. 22.3).

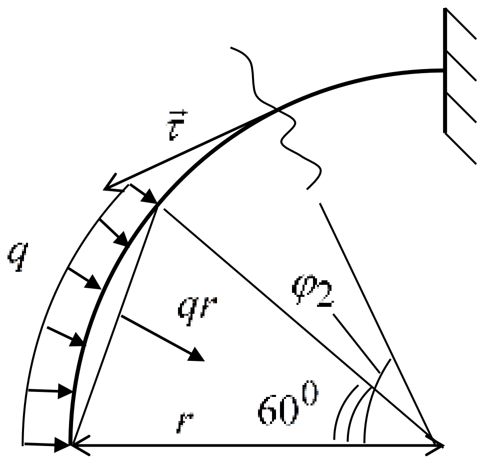

Fig. 22.3

$$N_{\varphi_2} = -q \cdot r \cdot \sin\left(\varphi_2 - 30^0\right) = -20 \cdot \sin\left(\varphi_2 - 30^0\right);$$

$$N_{\varphi_2}\left(60^0\right) = -20 \cdot 0{,}5 = -10 \text{ kN};$$

$$N_{\varphi_2}\left(90^0\right) = -20 \cdot 0{,}866 = -17{,}32 \text{ kN}.$$

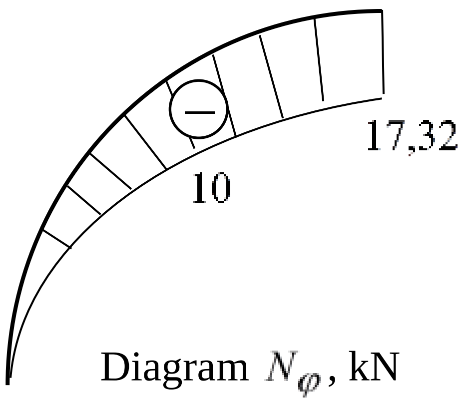

Diagram N_φ, kN

Fig. 22.4

On beginning

22.2. The building the diagram of shear force.

Let's turn to the building of the diagram of shearing forces Q_φ (Fig. 22.7). For this purpose, the forces are applied to the beam will be projected to the normal, which is located along the radius at the transversal section.

I portion. $0 \le \varphi_1 \le 60^0$ (fig. 22.5).

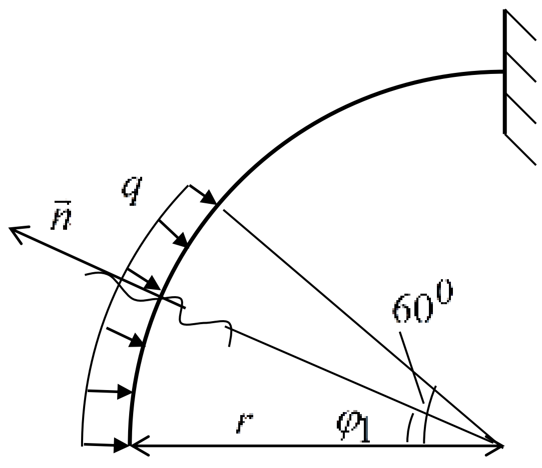

Fig. 22.5

$$Q_{\varphi_1}=-2q\cdot r\cdot\sin\frac{\varphi_1}{2}\cos\frac{\varphi_1}{2}=-q\cdot r\cdot\sin\varphi_1=-20\cdot\sin\varphi_1;$$

$$Q_{\varphi_1}(0)=0\ \text{kN};\quad Q_{\varphi_1}\left(60^0\right)=-20\cdot 0{,}866=-17{,}32\ \text{kN}.$$

II portion. $60^0\le\varphi_2\le 90^0$ (fig. 22.6).

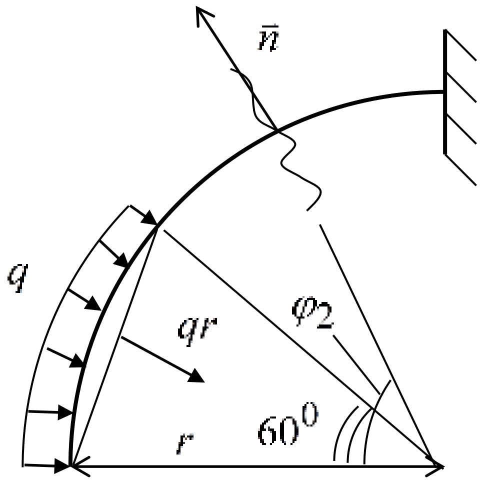

Fig. 22.6

$$Q_{\varphi_2}=-q\cdot r\cdot\cos\left(\varphi_2-30^0\right)=-20\cdot\cos\left(\varphi_2-30^0\right);$$
$$Q_{\varphi_2}\left(60^0\right)=-20\cdot 0{,}866=-17{,}32\ \text{kN};$$

$$Q_{\varphi_2}\left(90^0\right)=-20\cdot 0{,}5=-10\ \text{kN}.$$

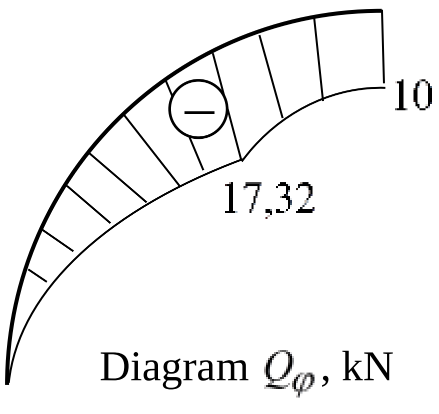

Diagram Q_{φ}, kN

Fig. 22.7

On beginning

21.3. The building the diagram of bend moment.

Finally, consider the procedure for building a bending moment diagram. We will use the following rule of signs: ***if the force tries to give the beam even more curvature, then the moment of such force will be considered positive if the force is trying to straighten the beam, then the moment from it is negative***.

Let us build the bending moment's diagram M_{φ} (fig. 22.10):

I portion. $0 \le \varphi_1 \le 60^0$ (fig. 22.8).

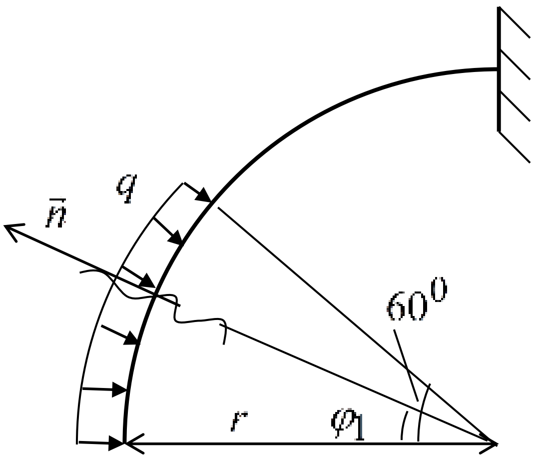

Fig. 22.8

$$M_{\varphi_1} = 2q \cdot r \cdot \left(\sin \frac{\varphi_1}{2} \right) \cdot r \cdot \sin \frac{\varphi_1}{2} = q \cdot r^2 \cdot (1 - \cos \varphi_1),$$

or

$$M_{\varphi_1} = 40 \cdot (1 - \cos \varphi_1);$$

$$M_{\varphi_1}(0) = 0 \text{ kNm}; \qquad M_{\varphi_1}\left(60^0\right) = 40 \cdot 0{,}5 = 20 \text{ kNm}.$$

II portion. $60^0 \le \varphi_2 \le 90^0$ (fig. 22.9).

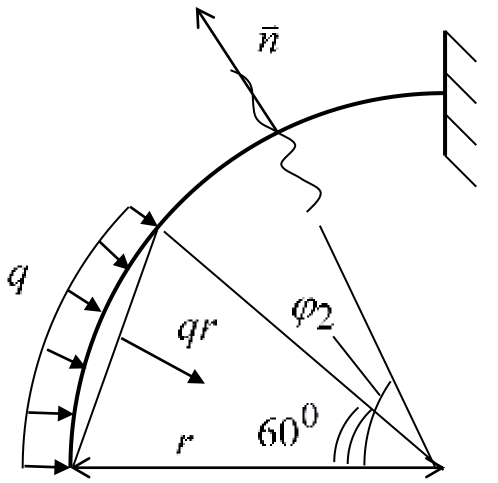

Fig. 22.9

$$M_{\varphi_2} = q \cdot r \cdot r \cdot \sin\left(\varphi_2 - 30^0\right) = q \cdot r^2 \cdot \sin\left(\varphi_2 - 30^0\right),$$

or

$$M_{\varphi_2} = 40 \cdot \sin\left(\varphi_2 - 30^0\right)$$

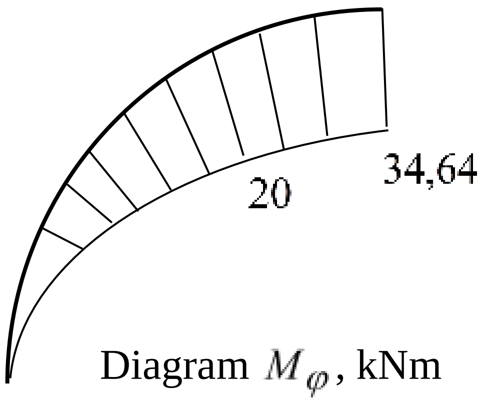

Diagram M_φ, kNm

Fig. 22.10

$$M_{\varphi_2}\left(60^0\right) = 40 \cdot 0{,}5 = 20 \text{ kNm};$$

$$M_{\varphi_2}\left(90^0\right) = 40 \cdot 0{,}866 = 34{,}64 \text{ kNm}.$$

On beginning