Lecture 21
THE CALCULATION OF CURVETTED BEAMS

Plan

1. General theory.
2. The definition reaction supports of curvetted beam, which is loaded by vertical forces.
3. The definition reaction supports of curvetted beam, on which acting the arbitrary load.

21.1. General theory.

The systems which have curvilinear or polygonal line image are called **arched constructions**. In mechanics of materials and structures arched construction is called curvetted beam (fig. 21.1).

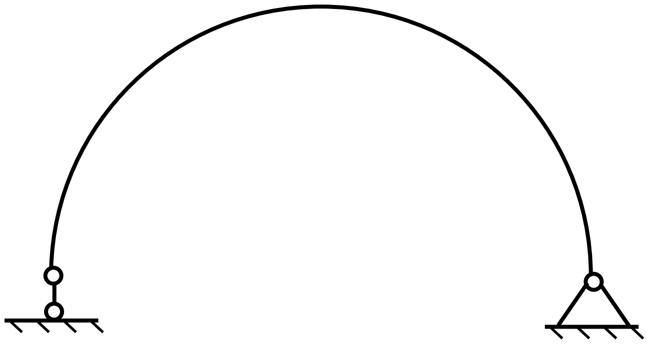

Fig.21.1

In the supports of curvetted beam from the vertical load there are oblique reactions, which are usually directed inside the span (fig. 21.2).

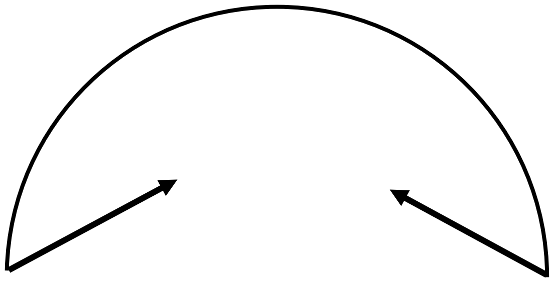

Fig. 21.2

Arched constructions compared with beam are sometimes more economically advantageous. In this regard, they began used for building bridges at the time of Kulibina I P and Euler.

Calculation of curvetted beams as beams and frames begins with the determination of reactions of supports. Let's consider several examples of determining the support reactions of curvetted beams from different types of acting an external load.

Let's start with consideration of the case when several vertical concentrated forces are applied to the arch.

On beginning

21.2. The definition reaction supports of curvetted beam, which is loaded by vertical forces.

Let a given arch in the form of a semicircle, which is simple supported curvetted beam (fig. 21.3). Two vertical, concentrated forces $F_1 = 5$ kN and $F_2 = 10$ kN are applied to the arch.

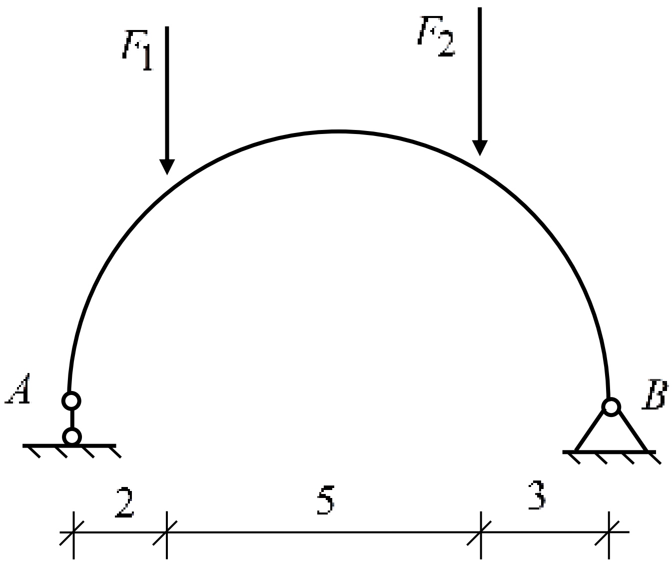

Fig. 21.3

Determine the reactions of the given curvetted beam from the equations of the static.

We reject the supports and replace them with the corresponding reactions R_A, H_B and V_B (fig. 21.4).

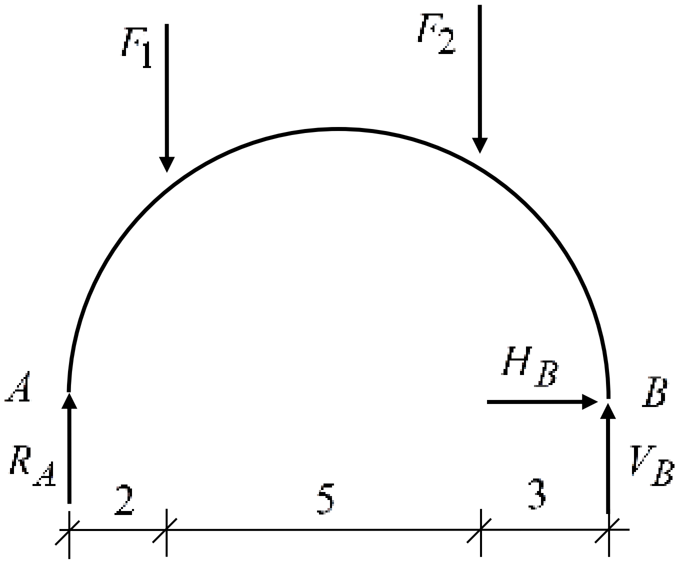

Fig. 21.4

Then from the equations of the static we will obtain:

1. $\sum F_{x_i} = 0$: $H_B = 0$;

2. $\sum M_{A_i} = 0$: $V_B \cdot 10 - F_1 \cdot 2 - F_2 \cdot 7 = 0$,

where is:

$$V_B = \frac{F_1 \cdot 2 + F_2 \cdot 7}{10}$$

or

$$V_B = \frac{5 \cdot 2 + 10 \cdot 7}{10} = 8 \text{ кН};$$

3. $\sum M_{B_i} = 0$: $-R_A \cdot 10 + F_1 \cdot 8 + F_2 \cdot 3 = 0$,

where is:

$$R_A = \frac{F_1 \cdot 8 + F_2 \cdot 3}{10}$$

or

$$R_A = \frac{5 \cdot 8 + 10 \cdot 3}{10} = 7 \text{ кН}.$$

Make sure the reactions are found right. To do this, we write the equation:

$$\sum F_{y_i} = 0: \quad R_A + V_B - F_1 - F_2 = 7 + 8 - 5 - 10 = 15 - 15 = 0.$$

Consequently, the reaction of the supports of the curvetted beam is found correctly.

Now it is advisable to consider a more interesting case when the force to the curvetted beam is applied at an arbitrary angle.

On beginning

21.3. The definition reaction supports of curvetted beam, on which acting the arbitrary load.

On fig. 21.5 the curvetted beam is represented in the form of a quarter of a circle with radius which is 5 m. This curvetted beam is simply supported.

The force of 10 kN acts to the curvetted beam along the normal at an angle 30^0 to the horizontal.

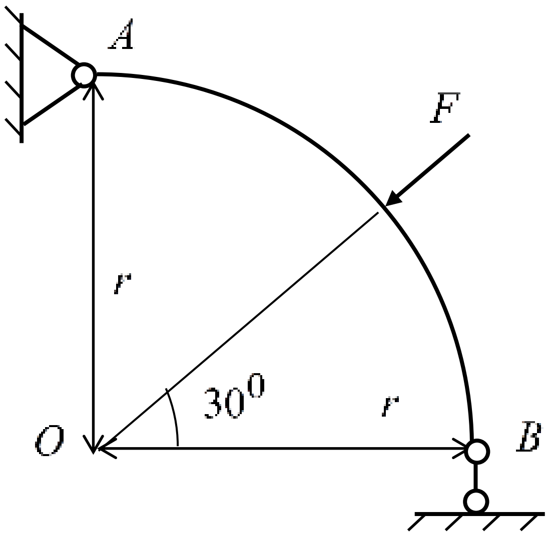

Fig. 21.5

Let us release the curvetted beam from the supports and it is loaded by corresponding reactions (Fig. 21.6).

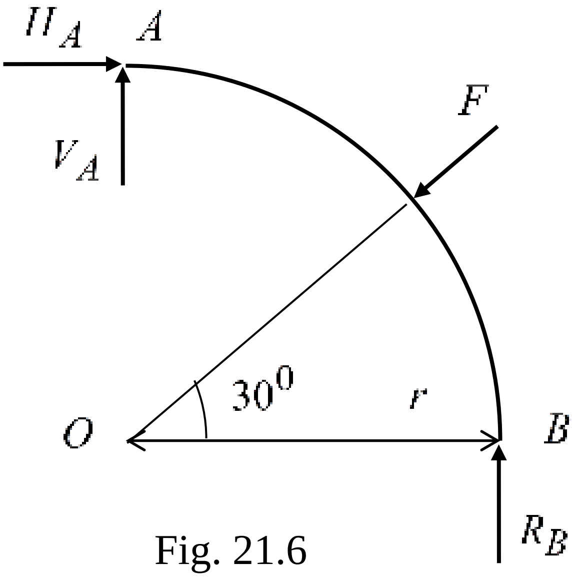

Fig. 21.6

Let's write down the equation of equilibrium for the given curvetted beam:

1. $\sum F_{x_i} = 0$: $\quad H_A - F \cdot \cos 30^0 = 0$,

where is:

$$H_A = F \cdot \cos 30^0,$$

or

$$H_A = 10 \cdot 0{,}866 = 8{,}66 \text{ кН};$$

2. $\sum M_{A_i} = 0$: $\quad R_B \cdot r - F \cdot r \cdot \sin 60^0 = 0$,

where is:

$$R_B = F \cdot \sin 60^0$$

or

$$R_B = 10 \cdot 0{,}866 = 8{,}66 \text{ кН};$$

3. $\sum M_{B_i} = 0$: $\quad -V_A \cdot r + F \cdot r \cdot \sin 30^0 - H_A \cdot r = 0$,

where is:

$$V_A = F \cdot \sin 30^0 - H_A$$

or

$$V_A = 10 \cdot 0{,}5 - 8{,}66 = -3{,}66 \text{ кН}.$$

Make sure the reactions are found correctly. To do this, we write the equation:

$$\sum F_{y_i} = 0: \quad V_A + R_B - F \cdot \sin 30^0 = -3{,}66 + 8{,}66 - 10 \cdot 0{,}5 =$$

$$= -8{,}66 + 8{,}66 = 0.$$

Hence, the reaction of the supports of the curvetted beam is found correctly.

On beginning